

Linear Equations, Inequalities and Systems: math lesson, Grade 11 – download the PDF

A phone plan with a flat fee plus a price per gigabyte, a delivery truck that must respect weight limits, a recipe that has to hit a calorie target: all of these are modeled with **linear relationships**. In this chapter you will sharpen your skills with lines and slopes, solve equations and inequalities with absolute value, handle compound inequalities, and then team up several equations into **systems** that you can solve by hand, by graphing, or with matrices. You will even use them to make the best possible decision under constraints.

1. Linear equations and slope

A linear equation in one variable, such as $3x - 7 = 11$, is solved by undoing operations in reverse order. When fractions appear, multiply every term by the least common denominator first. A linear equation in two variables describes a **line** in the coordinate plane.

D Slope

The slope of the line through (x_1, y_1) and (x_2, y_2) , with $x_1 \neq x_2$, is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}.$$

A positive slope rises from left to right, a negative slope falls, a horizontal line has slope **0**, and a vertical line has no slope (it is undefined).

A line can be written in three common forms: **slope-intercept form** $y = mx + b$ (slope m , y-intercept b), **point-slope form** $y - y_1 = m(x - x_1)$, and **standard form** $Ax + By = C$.

P Parallel and perpendicular lines

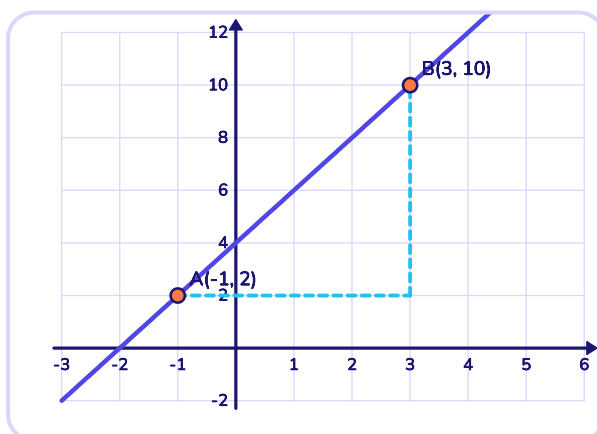
Two non-vertical lines are **parallel** when they have the same slope. They are **perpendicular** when the product of their slopes is -1 , that is, $m_2 = -\frac{1}{m_1}$.

E Example 1: equation of a line

Find the equation of the line through $A(-1, 2)$ and $B(3, 10)$.

The slope is $m = \frac{10 - 2}{3 - (-1)} = \frac{8}{4} = 2$. Use point-slope form with A :

$y - 2 = 2(x + 1)$, so $y = 2x + 4$. Check with B : $2(3) + 4 = 10$. It works.



E**Example 2: a perpendicular line**

Find the line perpendicular to $y = 2x + 4$ that passes through $(4, 1)$.

The new slope is the opposite reciprocal, $-\frac{1}{2}$. Then $y - 1 = -\frac{1}{2}(x - 4)$, so $y = -\frac{1}{2}x + 3$. Check: at $x = 4$, $-2 + 3 = 1$.

E**Example 3: an equation with fractions**

Solve $\frac{x + 3}{4} - \frac{x - 1}{6} = 2$.

The least common denominator is **12**. Multiply every term:

$3(x + 3) - 2(x - 1) = 24$. Distribute: $3x + 9 - 2x + 2 = 24$, so $x + 11 = 24$ and $x = 13$. Check: $\frac{16}{4} - \frac{12}{6} = 4 - 2 = 2$.

2. Absolute value equations and inequalities

The absolute value $|a|$ is the distance between a and **0** on a number line, so it is never negative. This distance idea explains every rule below.

P**Absolute value rules**

For a number $c > 0$:

- $|u| = c$ means $u = c$ or $u = -c$;
- $|u| < c$ means $-c < u < c$ (an **and** statement);
- $|u| > c$ means $u < -c$ or $u > c$ (an **or** statement).

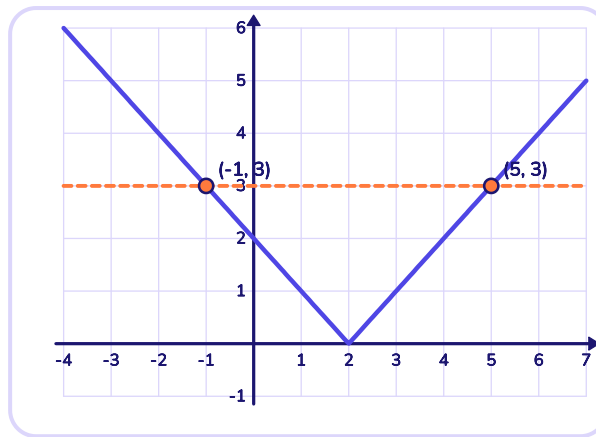
If $c < 0$, then $|u| = c$ and $|u| < c$ have no solution, while $|u| > c$ is true for every real number.

M**Method**

1. Isolate the absolute value expression on one side.
2. Check the sign of the number on the other side.
3. Split into the two cases given by the rules above.
4. Solve each case and check your answers in the original statement.

E**Example 4: an equation**

Solve $|x - 2| = 3$. Either $x - 2 = 3$ or $x - 2 = -3$, so $x = 5$ or $x = -1$. On the graph, these are the two points where the V-shaped graph of $y = |x - 2|$ meets the line $y = 3$.



E Example 5: an inequality with an and

Solve $2|3x - 1| + 4 \leq 18$. Subtract 4: $2|3x - 1| \leq 14$. Divide by 2: $|3x - 1| \leq 7$. Then $-7 \leq 3x - 1 \leq 7$, so $-6 \leq 3x \leq 8$ and $-2 \leq x \leq \frac{8}{3}$.

E Example 6: an inequality with an or

Solve $|2x + 5| > 9$. Then $2x + 5 > 9$ or $2x + 5 < -9$. The first gives $x > 2$, the second gives $x < -7$. The solution set is $x < -7$ or $x > 2$.



Common mistake

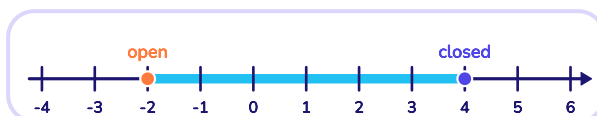
Do not split the equation before isolating the absolute value. In $2|x| + 4 = 10$, first get $|x| = 3$; only then write $x = 3$ or $x = -3$. Also remember to reverse the inequality sign whenever you multiply or divide by a negative number.

3. Compound inequalities

A **compound inequality** joins two inequalities with the word **and** or the word **or**. An **and** compound requires both parts to be true, so its solution is the **intersection** of the two solution sets. An **or** compound requires at least one part to be true, so its solution is the **union**.

E Example 7: a three-part inequality

Solve $-3 < 2x + 1 \leq 9$. Do the same operation on all three parts. Subtract 1: $-4 < 2x \leq 8$. Divide by 2: $-2 < x \leq 4$. In interval notation, this is $(-2, 4]$.



On the number line, an **open** circle marks an endpoint that is not included (strict inequality $<$ or $>$), and a **closed** circle marks an included endpoint ($\leq$ or $\geq$). The shaded segment above is the solution of Example 7.

E Example 8: an or statement

Solve $3x - 2 < -8$ or $5x + 1 \geq 16$. The first inequality gives $3x < -6$, so $x < -2$. The second gives $5x \geq 15$, so $x \geq 3$. The solution is $(-\infty, -2) \cup [3, \infty)$, two separate pieces of the number line.

! Zyro's tip

On my home planet we say: “and” squeezes, “or” stretches. An **and** statement usually gives one trimmed-down interval, while an **or** statement can give two pieces that point in opposite directions.

4. Systems of two equations

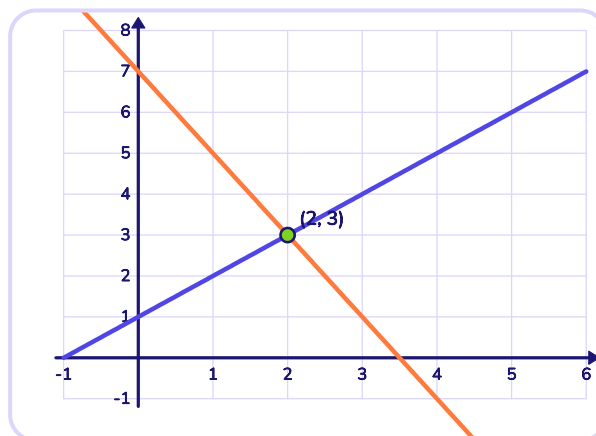
A **system of linear equations** is a set of equations that must hold at the same time. A solution is an ordered pair that makes every equation true. Geometrically, it is a point shared by all the lines. Two lines can cross at one point (one solution), be parallel (no solution, an **inconsistent** system), or be the same line (infinitely many solutions, a **dependent** system).

M**Three methods**

1. **Graphing:** draw both lines and read the intersection (good for a quick picture, less precise).
2. **Substitution:** solve one equation for one variable and substitute into the other (best when a variable has coefficient **1**).
3. **Elimination:** multiply equations so that a variable has opposite coefficients, then add (best for standard form).

E**Example 9: substitution**

Solve $y = x + 1$ and $y = -2x + 7$. Set the right sides equal: $x + 1 = -2x + 7$, so $3x = 6$ and $x = 2$. Then $y = 2 + 1 = 3$. The solution is $(2, 3)$, exactly where the two lines cross in the graph below.



E**Example 10: elimination**

Solve $3x + 4y = 10$ and $5x - 2y = 8$. Multiply the second equation by 2:

$10x - 4y = 16$. Add it to the first equation: $13x = 26$, so $x = 2$. Substitute into the first equation: $6 + 4y = 10$, so $y = 1$. Check in the second equation: $10 - 2 = 8$.

If elimination makes every variable disappear, look at what is left. A false statement such as $0 = 4$ means **no solution**; a true statement such as $0 = 0$ means **infinitely many solutions**. For instance, $2x - 4y = 6$ and $x - 2y = 5$ become $2x - 4y = 6$ and $2x - 4y = 10$, which contradict each other: the lines are parallel.

5. Systems of three equations

With three unknowns x, y, z , each equation describes a plane in space. The usual strategy is to use elimination twice to remove the **same** variable from two different pairs of equations. That leaves a system of two equations in two unknowns, which you already know how to solve. Then you back-substitute to find the last variable.

E**Example 11: three unknowns**

Solve $x + y + z = 6$, $2x - y + z = 3$, $x + 2y - z = 2$.

Add the first and third equations: $2x + 3y = 8$. Add the second and third:

$3x + y = 5$, so $y = 5 - 3x$. Substitute: $2x + 15 - 9x = 8$, so $-7x = -7$ and $x = 1$. Then $y = 2$, and from the first equation $z = 6 - 1 - 2 = 3$. The solution is $(1, 2, 3)$.

Check the second equation: $2 - 2 + 3 = 3$, and the third: $1 + 4 - 3 = 2$.



Stay organized

Number your equations, write down which variable you are eliminating, and always check the final triple in **all three** original equations. A single sign error early on spreads everywhere.

6. Linear programming

Linear programming finds the best value (largest profit or smallest cost) of a linear **objective function** when the variables must satisfy linear **constraints**. The set of points that satisfy all constraints is the **feasible region**.



Corner point principle

If an objective function has a maximum or a minimum on a feasible region that is a bounded polygon, that optimal value is reached at a **vertex** (corner point) of the region.



Method

1. Define the variables and write the constraints, including $x \geq 0$ and $y \geq 0$ when quantities cannot be negative.
2. Graph the constraints and shade the feasible region.
3. Find each vertex by solving a small system.
4. Evaluate the objective function at every vertex and pick the largest or smallest value.

E

Example 12: maximizing profit

A bakery makes x trays of muffins and y trays of scones. The oven allows

$2x + y \leq 14$ hours and the staff allows $x + 2y \leq 16$ hours, with $x \geq 0$ and $y \geq 0$.

The profit is $P = 30x + 20y$ dollars. Find the maximum profit.

The vertices of the shaded region are $(0, 0)$, $(7, 0)$, $(0, 8)$, and the intersection of $2x + y = 14$ with $x + 2y = 16$, which is $(4, 6)$.



Vertex

 $P = 30x + 20y$ $(0, 0)$ **0** $(7, 0)$ **210** $(4, 6)$ **240** $(0, 8)$ **160**

The largest value is **240**, so the bakery earns a maximum profit of **240** dollars by making **4** trays of muffins and **6** trays of scones.

7. Matrices and solving systems

A **matrix** is a rectangular array of numbers. A system can be written compactly as $\mathbf{AX} = \mathbf{B}$, where $\mathbf{A}$ holds the coefficients, $\mathbf{X}$ the unknowns, and $\mathbf{B}$ the constants. For example, $3x + 2y = 8$ and $x + 4y = 6$ become

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}.$$

D Determinant and inverse

For $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the **determinant** is $\det \mathbf{A} = ad - bc$. If $\det \mathbf{A} \neq 0$, the inverse exists:

$$\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

If $\det \mathbf{A} = 0$, the system has either no solution or infinitely many.

E Example 13: solving with an inverse

For the system above, $\det \mathbf{A} = 3 \cdot 4 - 2 \cdot 1 = 10$, so

$$\mathbf{A}^{-1} = \frac{1}{10} \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}.$$

Then $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \frac{1}{10} \begin{bmatrix} 4 \cdot 8 - 2 \cdot 6 \\ -8 + 3 \cdot 6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$. The solution is $x = 2$, $y = 1$. Check: $3(2) + 2(1) = 8$ and $2 + 4(1) = 6$.

The order matters: always multiply $\mathbf{A}^{-1}$ on the **left** of $\mathbf{B}$. For three equations, a graphing calculator or software can find the inverse or the reduced row echelon form of the augmented matrix; your job is to set up the matrices correctly and to interpret the result in context.

Key takeaways

- Slope is rise over run; parallel lines share a slope, perpendicular slopes multiply to -1 .
- $|u| < c$ gives an **and** statement, $|u| > c$ gives an **or** statement; isolate the absolute value first.
- Flip an inequality sign when multiplying or dividing by a negative number.
- A two-variable system has one, none, or infinitely many solutions; use graphing, substitution, or elimination.
- For three unknowns, eliminate the same variable twice to get a smaller system.
- In linear programming, test every vertex of the feasible region in the objective function.
- Write a system as $AX = B$; when $\det A \neq 0$, $X = A^{-1}B$.



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