

Linear Inequalities: math lesson, Grade 9 – download the PDF

An equation says two quantities are equal. An inequality says one quantity is bigger, smaller, or at least as big as another. Inequalities describe limits, budgets, speed caps, safe temperatures, and minimum scores, so you meet them everywhere. In this chapter you will learn to read the symbols, solve inequalities, flip the sign at exactly the right moment, graph answers on a number line, combine inequalities with *and* and *or*, work with absolute value, and turn real situations into math.

1. Inequality symbols

There are four main symbols, plus one that says two numbers are different. Notice that the symbol always opens toward the larger number.

Symbol	Meaning	Typical words	Example
$<$	less than	fewer than, below, under	$3 < 8$
$>$	greater than	more than, above, over, exceeds	$9 > 2$
$\leq$	less than or equal to	at most, no more than, maximum	$x \leq 5$
$\geq$	greater than or equal to	at least, no less than, minimum	$x \geq -2$
$\neq$	not equal to	different from	$x \neq 0$

D Linear inequality

A **linear inequality** in one variable is a statement such as $(ax+b), \leq, \geq)$ in which the variable x appears only to the first power. A **solution** is any number that makes the statement true. The **solution set** is the collection of all solutions.

An equation like $x = 4$ has one solution, but an inequality like $x > 4$ has infinitely many: 4.1, 5, 17.5, and 1,000 all work, while 4 itself does not.

E Example 1: testing a number

Is 5 a solution of $2x - 3 \geq 7$? Replace x by 5: $2(5) - 3 = 7$, and $7 \geq 7$ is true, so yes. Is 4 a solution? $2(4) - 3 = 5$, and $5 \geq 7$ is false, so no.

2. Solving one-step and multi-step inequalities

You solve an inequality almost exactly like an equation: you undo operations in reverse order, doing the same thing to both sides. Adding or subtracting the same number on both sides never changes the solution set. Multiplying or dividing by a *positive* number does not change it either.

M Method: solving a multi-step inequality

1. Clear parentheses with the distributive property and combine like terms on each side.
2. Collect the variable terms on one side and the numbers on the other by adding or subtracting.
3. Divide (or multiply) to isolate x . If the number is negative, reverse the inequality sign (see Part 3).
4. Check with one number inside the solution set and one outside it.

E Example 2: two steps

Solve $3x + 5 < 20$. Subtract 5: $3x < 15$. Divide by 3: $x < 5$. Check: $x = 4$ gives $17 < 20$, true; $x = 6$ gives $23 < 20$, false.

E Example 3: variables on both sides

Solve $4(x - 2) + 3 \geq 7x - 14$. Distribute: $4x - 8 + 3 \geq 7x - 14$, so $4x - 5 \geq 7x - 14$. Subtract $7x$: $-3x - 5 \geq -14$. Add 5: $-3x \geq -9$. Divide by -3 and reverse the sign: $x \leq 3$. Check with $x = 0$: $-5 \geq -14$, true.

3. Reversing the inequality sign

Take the true statement $2 < 5$. Multiply both sides by -1 : you get -2 and -5 , and -2 is actually *greater* than -5 . Multiplying by a negative number flips the order on the number line, so the symbol must flip too.

P Rule for negative numbers

If you multiply or divide both sides of an inequality by a **negative** number, you must **reverse** the inequality symbol: $<$ becomes $>$, $\leq$ becomes $\geq$, and vice versa.

E Example 4: dividing by a negative

Solve $5 - 3x \leq 17$. Subtract 5: $-3x \leq 12$. Divide by -3 and flip: $x \geq -4$. Check $x = -4$: $5 + 12 = 17 \leq 17$, true. Check $x = 0$: $5 \leq 17$, true. Check $x = -6$: $5 + 18 = 23 \leq 17$, false.

A Common mistake

Do **not** flip the sign when you add or subtract a negative number. Only multiplication or division by a negative number flips it. Also remember that $-x > 2$ means $-1 \cdot x > 2$, so it becomes $x < -2$.

4. Graphing on a number line

A graph shows the whole solution set at a glance. Use a circle at the boundary number and shade the side that holds the solutions.

- **Open circle** for $<$ and $>$: the boundary number is not a solution.

- **Closed (filled) circle** for $\leq$ and $\geq$: the boundary number is a solution.
- Shade **left** for “less than” and **right** for “greater than.”



The first graph shows $x < 2$; the second shows $x \geq -1$. You can also write solution sets in interval notation, such as $(-\infty, 2)$ and $[-1, \infty)$, where a parenthesis means “not included” and a bracket means “included.”

Zyro's tip

On my home planet we say: “the symbol points at the small number.” In $x < 2$ the small end is next to x , so x lives on the small side of 2.

5. Compound inequalities

A **compound inequality** joins two inequalities with *and* or *or*.

- **And** (intersection): both statements must be true at once. The graph is the overlap, often a segment such as $\setminus(-2$
- **Or** (union): at least one statement must be true. The graph is usually two rays pointing away from each other.

Example 5: an “and” inequality

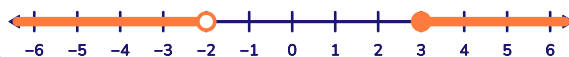
Solve $-3 < 2x + 1 \leq 9$. Whatever you do to the middle, do to all three parts.

Subtract 1: $-4 < 2x \leq 8$. Divide by 2: $\setminus(-2$



E Example 6: an “or” inequality

Solve $x - 1 < -3$ or $2x \geq 8$. The first gives $x < -2$. The second gives $x \geq 4$. The solution set is $x < -2$ or $x \geq 4$, two separate rays. Notice the graph cannot be written as one chain.



Sometimes “and” leaves nothing: $x > 5$ and $x < 2$ has **no solution**, because no number is both bigger than 5 and smaller than 2. And sometimes “or” covers everything.



Common mistake

If you divide all three parts of a chain by a negative number, the sign flips in both places *and* the order of the chain reverses: $4 \leq -2x < 10$ becomes $-2 \geq x > -5$, which is written $-5 < x \leq -2$.

6. Absolute value equations

The **absolute value** of a number is its distance from 0 on the number line, so it is never negative: $|7| = 7$ and $|-7| = 7$.

P Solving $|A| = c$

If $c > 0$, then $|A| = c$ means $A = c$ or $A = -c$. If $c = 0$, then $A = 0$. If $c < 0$, there is **no solution**, because a distance cannot be negative.

M Method

1. Isolate the absolute value expression on one side.
2. Check the other side: if it is negative, stop, there is no solution.
3. Write the two cases $A = c$ and $A = -c$ and solve each one.
4. Check both answers in the original equation.

E Example 7: isolate first

Solve $3|x + 2| - 4 = 11$. Add 4: $3|x + 2| = 15$. Divide by 3: $|x + 2| = 5$. Case 1: $x + 2 = 5$, so $x = 3$. Case 2: $x + 2 = -5$, so $x = -7$. Check: $3|5| - 4 = 11$ and $3|-5| - 4 = 11$. Both work.

7. Absolute value inequalities

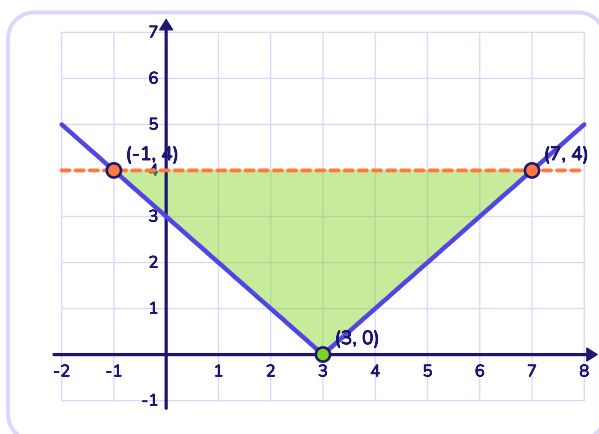
Because $|A|$ is a distance, $|A| > c$ means “farther than c from zero.”

P Two patterns (with $c > 0$)

\\(|A| and inequality). $|A| > c$ is the same as $A < -c$ or $A > c$ (an or inequality). The same holds for $\leq$ and $\geq$, with closed circles.

E Example 8: “less than” gives “and”

Solve $|x - 3| \leq 4$. Write $-4 \leq x - 3 \leq 4$. Add 3 to every part: $-1 \leq x \leq 7$. In the graph of $y = |x - 3|$, the points below the dashed line $y = 4$ are exactly the x -values between -1 and 7.



E Example 9: “greater than” gives “or”

Solve $|2x + 1| > 7$. Then $2x + 1 > 7$ gives $x > 3$, and $2x + 1 < -7$ gives $x < -4$. So $x < -4$ or $x > 3$.

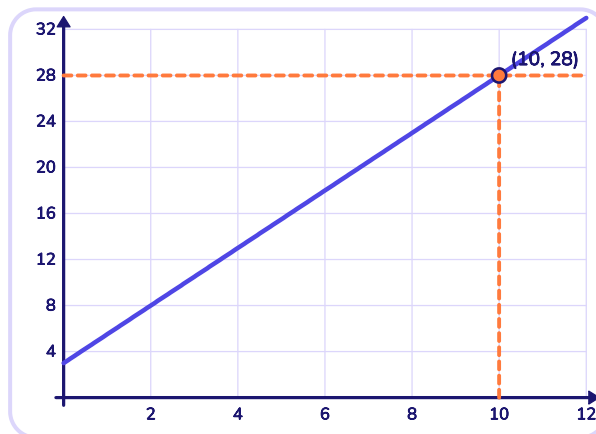
Two special cases: $|x| < -2$ has **no solution** (a distance is never below -2), while $|x| > -2$ is true for **every real number**.

8. Modeling real-world situations

Translate the words one piece at a time: choose a variable, identify the fixed amount and the amount that grows, and decide which symbol the key phrase means (“at most” is $\leq$, “at least” is $\geq$, “within” is an absolute value). Then solve, and answer in a complete sentence with units.

E Example 10: a taxi budget

A taxi charges \$3 to start plus \$2.50 per mile. You can spend at most \$28. Let m be the miles: $3 + 2.5m \leq 28$. Subtract 3: $2.5m \leq 25$. Divide by 2.5: $m \leq 10$. You can ride at most 10 miles (about 16.1 kilometers). The graph shows the fare line staying on or below the dashed budget line of \$28 until $m = 10$.



E Example 11: a tolerance

A recipe needs an oven at 350 °F, give or take 15 °F. The temperature T satisfies $|T - 350| \leq 15$, so $335 \leq T \leq 365$. In Celsius that is about 168 °C to 185 °C.

Whenever the unknown is a count (people, muffins, tickets), the answer must make sense: round in the direction that keeps the condition true, and use whole numbers.

Key takeaways

- $<, >$ use open circles; $\leq, \geq$ use closed circles. “At most” means $\leq$; “at least” means $\geq$.
- Solve an inequality like an equation, but **reverse the sign** when you multiply or divide by a negative number.
- “And” gives an overlap (a segment); “or” gives a union (usually two rays).
- $|A| = c$ gives two cases, $A = c$ or $A = -c$; no solution if $c < 0$.
- $|A| < c$ becomes $A < c$ or $A > -c$.
- In a word problem, define the variable, write the inequality, solve, check, and answer with units.



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