

Solving Linear Equations: math lesson, Grade 8 – download the PDF

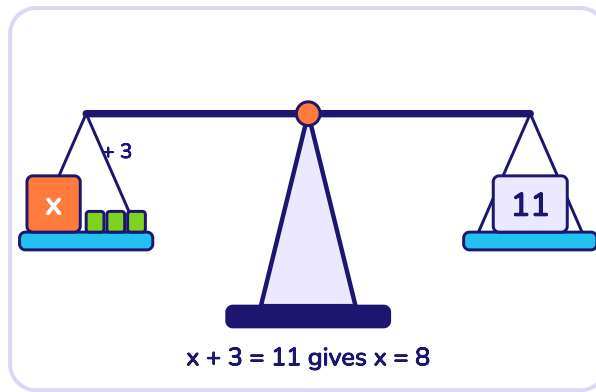
An equation is a promise that two expressions have the same value. Solving it means finding the number that keeps that promise. In Grade 8 you move from one-step puzzles to equations with parentheses, fractions, and variables on both sides, and you learn to tell when an equation has one answer, no answer, or endless answers. Every skill rests on one idea: do the same thing to both sides.

1. What is a linear equation?

D Definition

A **linear equation in one variable** is an equation that can be written in the form $ax + b = c$, where a , b , and c are numbers and $a \neq 0$. The variable appears only to the first power. A **solution** is a value of the variable that makes the equation true.

Think of a balance scale. The left pan holds $x + 3$ and the right pan holds **11**. The scale is level, so both pans weigh the same. If you take 3 away from each pan, the scale stays level, and the pan with the unknown box now balances **8**.



P

Property: the balance rule

You may add the same number to both sides, subtract the same number from both sides, multiply both sides by the same nonzero number, or divide both sides by the same nonzero number. The new equation has exactly the same solution as the old one.

A solution can be marked on a number line. For $x + 3 = 11$, the only point that works is **8**.



2. One-step and two-step equations

To isolate the variable, undo operations in the *reverse* order of the order of operations. Addition and subtraction are undone first, then multiplication and division.

M

Method: solve $ax + b = c$

1. Undo the addition or subtraction by working on both sides.
2. Undo the multiplication or division by the coefficient of x .
3. Check your answer by substituting it into the original equation.

E**Example 1: a two-step equation**

Solve $\frac{n}{4} - 5 = 3$.

$$\frac{n}{4} - 5 + 5 = 3 + 5 \Rightarrow \frac{n}{4} = 8 \Rightarrow n = 8 \times 4 = 32$$

Check: $\frac{32}{4} - 5 = 8 - 5 = 3$. It works.

**Watch out**

When the variable has a negative coefficient, as in $-3y = 21$, divide by -3 , not by 3 .
The answer is $y = -7$.

3. The distributive property and combining like terms

Parentheses are opened with the **distributive property**: $a(b + c) = ab + ac$. Every term inside gets multiplied, and a negative sign in front changes every sign inside. **Like terms** have the same variable part, so $5x$ and $-2x$ can be combined, and plain numbers can be combined with each other.

E**Example 2: parentheses and like terms**

Solve $3(2x - 5) + 4 = 19$.

$$6x - 15 + 4 = 19$$

$$6x - 11 = 19$$

$$6x = 30$$

$$x = 5$$

Check: $3(2 \cdot 5 - 5) + 4 = 3 \cdot 5 + 4 = 19$. It works.

**Watch out**

In $7 - 2(x + 1)$, the -2 multiplies both x and 1 : you get $7 - 2x - 2$, not $7 - 2x + 1$ and not $5x + 1$.

4. Variables on both sides

When x shows up on both sides, first simplify each side by itself, then gather all the variable terms on one side and all the numbers on the other. It is usually easiest to move the smaller variable term so that the coefficient stays positive.

M**Method: variables on both sides**

1. Distribute and combine like terms on each side.
2. Add or subtract a variable term so the variable is on one side only.
3. Add or subtract a number so the constants are on the other side.
4. Divide by the coefficient and check.

E**Example 3: both sides**

Solve $2(x + 4) - 3 = 5x - 4(x - 1)$.

$$2x + 8 - 3 = 5x - 4x + 4$$

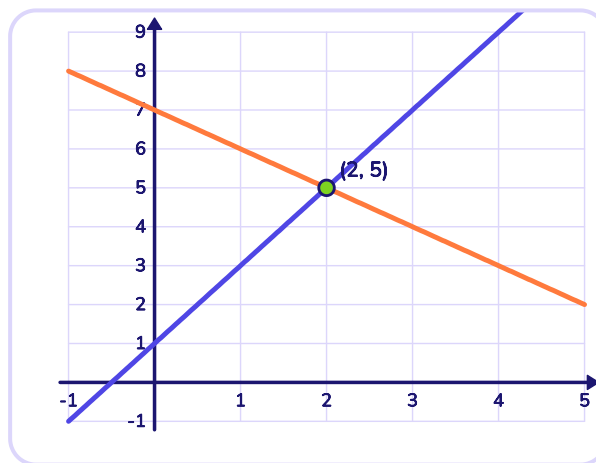
$$2x + 5 = x + 4$$

$$x + 5 = 4$$

$$x = -1$$

Check: left side $2(3) - 3 = 3$, right side $5(-1) - 4(-2) = -5 + 8 = 3$. Both sides equal **3**.

There is a picture behind this. Each side of an equation is a line when you graph y against x , and the solution is the x -coordinate of the point where the two lines meet. For $2x + 1 = -x + 7$ the lines cross at $(2, 5)$, so $x = 2$.

**5. Equations with fractions and decimals**

Rational coefficients are not harder, only messier. The trick is to clear them: multiply *every* term on both sides by a common denominator, or by a power of 10 for decimals. After that you have whole numbers only.

E**Example 4: clearing fractions**

Solve $\frac{2}{3}x - \frac{1}{4} = \frac{1}{2}x + \frac{5}{4}$.

The common denominator of 3, 4, and 2 is 12. Multiply every term by 12:

$$8x - 3 = 6x + 15 \Rightarrow 2x = 18 \Rightarrow x = 9$$

Check: left side $6 - 0.25 = 5.75$; right side $4.5 + 1.25 = 5.75$.

E**Example 5: clearing decimals**

Solve $0.4x + 1.5 = 0.15x + 4$. The most decimal places is 2, so multiply by 100:

$$40x + 150 = 15x + 400 \Rightarrow 25x = 250 \Rightarrow x = 10$$

Check: $0.4 \cdot 10 + 1.5 = 5.5$ and $0.15 \cdot 10 + 4 = 5.5$.

!**Zyro's tip**

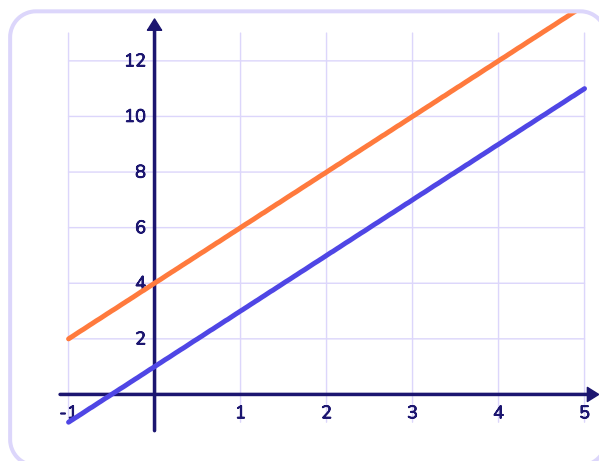
On my home planet we say: "Fractions are just numbers wearing costumes." Multiply by the common denominator and the costumes come off! Do not forget the terms without a fraction.

6. One solution, no solution, or infinitely many

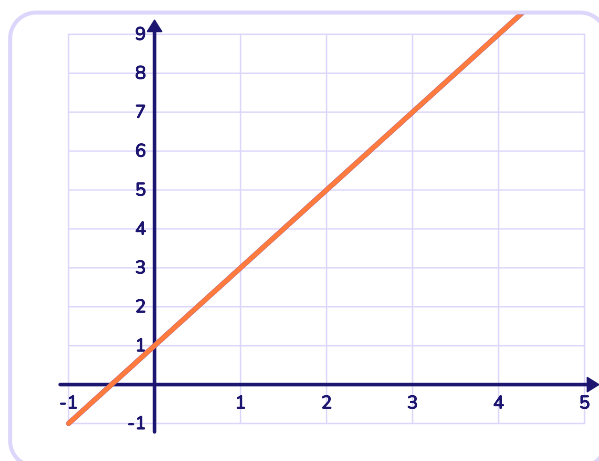
After you simplify, three things can happen.

Result after simplifying	Example	Number of solutions
$x = \text{a number}$	$2x + 5 = x + 9 \Rightarrow x = 4$	Exactly one
a false statement	$x + 3 = x + 7 \Rightarrow 3 = 7$	None
a true statement	$2(x + 1) = 2x + 2 \Rightarrow 2 = 2$	Infinitely many

For example, $4(x + 2) = 4x + 5$ becomes $4x + 8 = 4x + 5$, and subtracting $4x$ leaves $8 = 5$, which is false. No value of x can fix that. The two sides are parallel lines, such as $y = 2x + 1$ and $y = 2x + 4$.



On the other hand, $3(2x - 1) + 3 = 6x$ becomes $6x = 6x$, which is true for every number. Both sides describe the same line.





Watch out

Getting $0 = 0$ does *not* mean the answer is $x = 0$. It means every number works. Likewise, $8 = 5$ does not mean you made an arithmetic mistake, only that the equation has no solution.

7. Word problems and checking

To solve a word problem, name the unknown, write an equation from the story, solve it, and answer in a sentence with units. Always check the answer against the story, not only against your equation.



Example 6: comparing two gyms

Gym A charges a \$15 sign-up fee plus \$8 per month. Gym B has no fee but charges \$11 per month. After how many months do both cost the same?

$$15 + 8m = 11m \Rightarrow 15 = 3m \Rightarrow m = 5$$

Check: Gym A costs $15 + 40 = 55$ dollars and Gym B costs $11 \cdot 5 = 55$ dollars. After 5 months both cost \$55.

Key takeaways

- Whatever you do to one side of an equation, do to the other side.
- Undo addition and subtraction first, then multiplication and division.
- Distribute, then combine like terms, before moving anything across the equals sign.
- With variables on both sides, collect the variable on one side and the numbers on the other.
- Clear fractions by multiplying every term by the common denominator, and clear decimals with a power of 10.
- A false statement such as $8 = 5$ means no solution; a true one such as $0 = 0$ means infinitely many solutions.
- Always check by substituting your answer into the original equation.



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