

Limits, Probability, and Statistics: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Reading a limit from a table



For $x \neq 4$, $f(x) = \frac{(x-4)(x+4)}{x-4} = x+4$. So $f(3.9) = 7.9$, $f(3.99) = 7.99$, $f(4.01) = 8.01$, $f(4.1) = 8.1$.

The values approach 8 from both sides, so $\lim_{x \rightarrow 4} f(x) = 8$, even though $f(4)$ is undefined.

2 One-sided limits of a piecewise function



From the left use $x+2$: $1+2=3$. From the right use $6-x$: $6-1=5$.

Since $3 \neq 5$, the two-sided limit does **not** exist.

3 Two limits at infinity



a. The degrees are equal, so the limit is the ratio of leading coefficients: $\frac{5}{1} = 5$.

b. The numerator is constant while the denominator grows without bound, so the limit is 0.

4 Podium or committee?



a. Order matters: $P(6, 2) = 6 \cdot 5 = 30$.

b. Order does not matter: $C(6, 2) = \frac{6 \cdot 5}{2} = 15$.

Check: $P(6, 2) = 2! \cdot C(6, 2) = 30$.

5 Marbles without replacement



The first is red with probability $\frac{5}{8}$. Given that, 4 red remain among 7, so

$$P(\text{2nd red} \mid \text{1st red}) = \frac{4}{7}.$$

$$P = \frac{5}{8} \cdot \frac{4}{7} = \frac{20}{56} = \frac{5}{14} \approx 0.357.$$

6 Coin flips



This is binomial with $n = 4$, $p = 0.5$: $P(X = 3) = C(4, 3)(0.5)^3(0.5)^1 = 4 \cdot \frac{1}{16} = 0.25$.

7 Computing z-scores



a. $z = \frac{85 - 70}{10} = 1.5$.

b. $x = \mu + z\sigma = 70 + (-2)(10) = 50$.

8 Limit by factoring



Substitution gives $\frac{0}{0}$. Factor: $x^2 + x - 12 = (x + 4)(x - 3)$.

For $x \neq 3$ the fraction equals $x + 4$, so the limit is $3 + 4 = 7$.

9 Three limits at infinity



a. Equal degrees: $\frac{4}{2} = 2$.

b. Degree 2 below degree 3: the limit is 0.

c. Degree 3 above degree 2: dividing by x^2 gives $\frac{x}{1 + 4/x^2}$, which grows without bound, so there is no finite limit.

10 Arrangements of BANANA



There are 6 letters with A repeated 3 times and N repeated 2 times.

$$\frac{6!}{3!2!} = \frac{720}{12} = 60 \text{ arrangements.}$$

11 Choosing a committee



Choose the girls: $C(6, 3) = 20$. Choose the boys: $C(5, 2) = 10$.

By the counting principle, $20 \cdot 10 = 200$ committees.

12 Laptops and tablets



$$P(\text{tablet} \mid \text{laptop}) = \frac{P(\text{both})}{P(\text{laptop})} = \frac{0.45}{0.60} = 0.75.$$

The probability is 75%.

13 Free throws, exactly four



$$n = 5, p = 0.7, k = 4: P = C(5, 4)(0.7)^4(0.3)^1 = 5 \cdot 0.2401 \cdot 0.3 = 0.36015.$$

The probability is about 0.360, or 36%.

14 Reading scores



a. 85 and 115 are 1σ from the mean, so about 68%.

b. 130 is 2σ above the mean. About 95% lie within 2σ , so $\frac{100 - 95}{2} = 2.5\%$ lie above 130.

15 Standard deviation of a short list



Mean: $\frac{12 + 15 + 15 + 18 + 20}{5} = 16$. Deviations squared: 16, 1, 1, 4, 16, sum 38.

Population: $\sigma = \sqrt{38/5} = \sqrt{7.6} \approx 2.76$. Sample: $s = \sqrt{38/4} = \sqrt{9.5} \approx 3.08$ minutes.

16 Limit with a conjugate



Substitution gives $\frac{0}{0}$. Multiply top and bottom by the conjugate $\sqrt{x+9} + 3$:

$$\frac{(x+9)-9}{x(\sqrt{x+9}+3)} = \frac{x}{x(\sqrt{x+9}+3)} = \frac{1}{\sqrt{x+9}+3} \text{ for } x \neq 0.$$

As $x \rightarrow 0$ this tends to $\frac{1}{3+3} = \frac{1}{6}$.

17 Asymptotes of a rational function



Factor: $f(x) = \frac{3(x+2)}{(x-2)(x+1)}$. No factor cancels.

Vertical asymptotes: the denominator is 0 at $x = 2$ and $x = -1$, where the numerator is not 0.

Horizontal asymptote: the numerator degree 1 is below the denominator degree 2, so

$\lim_{x \rightarrow \pm\infty} f(x) = 0$ and the asymptote is $y = 0$.

18 PIN codes



First digit: 9 choices (1 to 9). Second digit: any unused digit, including 0, so 9 choices. Third: 8. Fourth: 7.

$$9 \cdot 9 \cdot 8 \cdot 7 = 4536 \text{ PINs.}$$

19 A medical screening test



$$P(\text{sick and } +) = 0.02 \cdot 0.95 = 0.019. \quad P(\text{healthy and } +) = 0.98 \cdot 0.04 = 0.0392.$$

$$P(+) = 0.019 + 0.0392 = 0.0582.$$

$$P(\text{sick} \mid +) = \frac{0.019}{0.0582} \approx 0.326. \text{ Only about 32.6\%, because the condition is rare.}$$

20 Guessing on a quiz



$$n = 10, p = 0.25.$$

a. $\mu = np = 2.5.$

b. $\sigma = \sqrt{10 \cdot 0.25 \cdot 0.75} = \sqrt{1.875} \approx 1.37.$

c. $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.75^{10} \approx 1 - 0.0563 = 0.9437.$

21 Bolts and tolerances



a. $z = \frac{49.2 - 50}{0.4} = -2$ and $z = \frac{50.8 - 50}{0.4} = 2$: about 95%.

b. $z = \frac{50.5 - 50}{0.4} = 1.25.$

c. $z = \frac{49.4 - 50}{0.4} = -1.5.$ It lies within 2σ of the mean, so it is not unusual.

22 Price and sales



$$\bar{x} = 4, \bar{y} = 66.$$

$$\sum(x - \bar{x})(y - \bar{y}) = (-2)(24) + (-1)(9) + 0 \cdot 4 + (1)(-11) + (2)(-26) = -120.$$

$$\sum(x - \bar{x})^2 = 10. \sum(y - \bar{y})^2 = 1470.$$

a. $m = \frac{-120}{10} = -12$, $b = 66 - (-12)(4) = 114$, so $\hat{y} = -12x + 114$.

b. $r = \frac{-120}{\sqrt{10 \cdot 1470}} \approx -0.99$: a very strong negative linear relationship.

c. Each extra dollar lowers predicted sales by about 12 units. At $x = 3.5$,

$$\hat{y} = -12(3.5) + 114 = 72 \text{ units.}$$



Test yourself: quick challenge for Grade 12

[Change level](#)

Level : **Grade 12**

6 random questions, instant feedback.

[Start the challenge](#)

Keep exploring with Zyro

 Math practice

Limits, Probability, and Statistics: math practice, Grade 12

 Math quizzes

Limits, Probability, and Statistics: math quiz, Grade 12

 Math tests

Limits, Probability, and Statistics: math test, Grade 12

 Math practice

Polynomial and Rational Functions Review: math practice, Grade 12

 Math practice

Exponential and Logarithmic Functions: math practice, Grade 12

 Math quizzes

Polynomial and Rational Functions Review: math quiz, Grade 12

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off