

Limits and Continuity: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Computing limits

/ 4 pts

(a) $\frac{(x-5)(x+5)}{x-5} = x + 5 \rightarrow 10$. (1 pt)

(b) Conjugate: $\frac{x}{x(\sqrt{x+4}+2)} = \frac{1}{\sqrt{x+4}+2} \rightarrow \frac{1}{4}$. (1 pt)

(c) Divide by x^2 : $\frac{7 - 1/x}{2 + 9/x^2} \rightarrow \frac{7}{2}$. (1 pt)

(d) $x^2 + x - 2 = (x+2)(x-1)$, so the quotient is $x+2 \rightarrow 3$. (1 pt)

2 A jump

/ 3 pts

(a) Left: $2 \cdot 3 - 1 = 5$. Right: $10 - 3 = 7$. (1 pt each, 2 pts)

(b) The one-sided limits differ, so the limit does not exist. (0.5 pt)

(c) $g(3) = 7$. The one-sided limits exist and differ: a jump discontinuity. (0.5 pt)

3 Asymptotes and a hole

/ 3 pts

Factor: $r(x) = \frac{2(x+3)}{(x-3)(x+3)} = \frac{2}{x-3}$ for $x \neq -3$.

(a) At $x = 3$ the denominator is 0 and the numerator is 12, so $x = 3$ is a vertical asymptote. (1 pt)

(b) At $x = -3$ the common factor cancels: the limit is $\frac{2}{-6} = -\frac{1}{3}$, so there is a hole at $(-3, -\frac{1}{3})$. (1 pt)

(c) $r(x) \rightarrow 0$ as $x \rightarrow \pm\infty$, so $y = 0$ is the horizontal asymptote. (1 pt)

4 Squeeze

/ 3 pts

Since $-1 \leq \sin x \leq 1$, $3 \leq 4 + \sin x \leq 5$; dividing by $x^2 > 0$ gives the inequality. (1 pt)

Both $\frac{3}{x^2}$ and $\frac{5}{x^2}$ tend to 0 as $x \rightarrow +\infty$. (1 pt)

By the squeeze theorem, the limit is 0. (1 pt)

5 Continuity at a point

/ 4 pts

(a) $x^2 - x - 6 = (x-3)(x+2)$, so $f(x) = x+2$ for $x \neq 3$ and the limit is 5. (2 pts)

(b) The limit exists (5) but differs from $f(3) = 1$: a removable discontinuity. (1 pt)

(c) Continuity requires $k = 5$. (1 pt)

6 Intermediate Value Theorem

/ 3 pts

(a) h is a polynomial, hence continuous. $h(1) = 1 + 2 - 5 = -2 < 0$ and $h(2) = 8 + 4 - 5 = 7 > 0$. By the IVT, there is a root in $(1, 2)$. (2 pts)

(b) $h(1) = -2 < 0$ and $h(1.5) = 1.375 > 0$, so a root lies in $(1, 1.5)$. (1 pt)



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