

Limits and Continuity: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Guess a limit from a table



$$f(2.9) = 6.8, f(2.99) = 6.98, f(2.999) = 6.998; f(3.1) = 7.2, f(3.01) = 7.02, f(3.001) = 7.002.$$

The values approach 7 from both sides, so we suggest $\lim_{x \rightarrow 3} (2x + 1) = 7$. (Substitution confirms it: $2 \cdot 3 + 1 = 7$.)

2 Direct substitution



A polynomial is continuous, so we substitute: $(-2)^2 + 3(-2) - 1 = 4 - 6 - 1 = -3$.

$$\lim_{x \rightarrow -2} (x^2 + 3x - 1) = -3.$$

3 Applying the limit laws



- a. $3 \cdot 4 + (-2) = 10$.
- b. $4 \cdot (-2) = -8$.
- c. The denominator limit is $-2 \neq 0$, so the limit is $\frac{4}{-2} = -2$.
- d. $4^2 - (-2) = 16 + 2 = 18$.

4 Reading limits on a graph



- a. On the left the curve approaches height 3; on the right it approaches height 2. So the left-hand limit is 3 and the right-hand limit is 2.
- b. The one-sided limits differ ($3 \neq 2$), so the limit does not exist.
- c. The filled point shows $h(1) = 5$.

5 Two sides of the absolute value



- a. For $x < 0$, $|x| = -x$, so $\frac{|x|}{x} = -1$. For $x > 0$, $|x| = x$, so the quotient is 1. The limits are -1 (left) and 1 (right); the two-sided limit does not exist.
- b. For $x < 2$, $|x - 2| = -(x - 2)$, so the quotient equals -1 . The limit is -1 .

6 A simple limit at infinity



Divide numerator and denominator by x : $\frac{4 + \frac{7}{x}}{2 - \frac{1}{x}}$. As $x \rightarrow +\infty$, $\frac{7}{x} \rightarrow 0$ and $\frac{1}{x} \rightarrow 0$, so the limit is $\frac{4}{2} = 2$.

7 True or false?



False. The limit ignores the value at 2. Take $f(x) = \frac{x^2 - 4}{x - 2}$: it is undefined at 2, but $f(x) = x + 2$ for $x \neq 2$, so $\lim_{x \rightarrow 2} f(x) = 4$.

8 Cancel a common factor



Substitution gives $\frac{0}{0}$. Factor: $x^2 + 5x + 6 = (x + 2)(x + 3)$. For $x \neq -3$ the quotient equals $x + 2$, so the limit is $-3 + 2 = -1$.

9 Rationalize the numerator



Substitution gives $\frac{0}{0}$. Multiply by the conjugate $\sqrt{x+9} + 3$: the numerator becomes $(x+9) - 9 = x$. So the quotient is $\frac{x}{x(\sqrt{x+9} + 3)} = \frac{1}{\sqrt{x+9} + 3}$ for $x \neq 0$. The limit is $\frac{1}{3+3} = \frac{1}{6}$.

10 Find all asymptotes



Horizontal: divide by x^2 : $\frac{6 + \frac{1}{x^2}}{3 - \frac{1}{x}} \rightarrow \frac{6}{3} = 2$ as $x \rightarrow \pm\infty$. So $y = 2$ is a horizontal asymptote.

Vertical: $3x^2 - x = x(3x - 1) = 0$ at $x = 0$ and $x = \frac{1}{3}$. The numerator equals 1 at $x = 0$ and $\frac{2}{3} + 1 = \frac{5}{3}$ at $x = \frac{1}{3}$; neither is 0. So $x = 0$ and $x = \frac{1}{3}$ are vertical asymptotes.

11 Square roots at infinity



We have $\sqrt{4x^2 + 1} = |x|\sqrt{4 + \frac{1}{x^2}}$.

For $x \rightarrow +\infty$: $|x| = x$, so the quotient is $\frac{\sqrt{4 + 1/x^2}}{1 + 3/x} \rightarrow \frac{2}{1} = 2$.

For $x \rightarrow -\infty$: $|x| = -x$, so the quotient is $-\frac{\sqrt{4 + 1/x^2}}{1 + 3/x} \rightarrow -2$.

12 Is it continuous at 2?



(1) $f(2) = 3 \cdot 2 - 1 = 5$ is defined.

(2) Left limit: $2^2 + 1 = 5$. Right limit: $3 \cdot 2 - 1 = 5$. They agree, so the limit exists and equals 5.

(3) The limit equals $f(2) = 5$. All three conditions hold, so f is continuous at 2.

13 Repair a hole



For $x \neq 4$, $f(x) = \frac{(x-4)(x+4)}{x-4} = x+4$, so $\lim_{x \rightarrow 4} f(x) = 8$. Continuity requires $k = f(4) = 8$.

14 Parking garage fees



$C(2) = 4 \cdot 2 = 8$ dollars. For $\setminus(1$

The one-sided limits differ, so C has a jump discontinuity at $t = 2$ (not continuous). The same jump of 4 dollars occurs at every integer $t = 1, 2, 3, 4, 5$ inside the interval.

15 Squeeze with a cosine

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For all $x \neq 0$, $-1 \leq \cos \frac{4}{x} \leq 1$. Multiplying by $x^2 \geq 0$: $-x^2 \leq x^2 \cos \frac{4}{x} \leq x^2$. Since $\lim_{x \rightarrow 0}(-x^2) = \lim_{x \rightarrow 0} x^2 = 0$, the squeeze theorem gives $\lim_{x \rightarrow 0} x^2 \cos \frac{4}{x} = 0$.

16 Squeeze at infinity

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a. For $x > 0$, $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$. Both bounds tend to 0, so the limit is 0.

b. $\frac{2x + \cos x}{x} = 2 + \frac{\cos x}{x}$. By the same squeeze, $\frac{\cos x}{x} \rightarrow 0$, so the limit is 2.

17 Locate three roots

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f is a polynomial, hence continuous. $f(-3) = -27 + 12 + 1 = -14$ and $f(-2) = -8 + 8 + 1 = 1$: opposite signs, so a root lies in $(-3, -2)$.

$f(0) = 1$ and $f(1) = 1 - 4 + 1 = -2$: a root lies in $(0, 1)$.

$f(1) = -2$ and $f(2) = 8 - 8 + 1 = 1$: a root lies in $(1, 2)$.

A cubic has at most three real roots, so these are all of them.

18 Classify the discontinuities

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The denominator factors: $x^2 - 3x + 2 = (x - 1)(x - 2)$, so f is undefined at $x = 1$ and $x = 2$. Also $f(x) = \frac{(x - 1)(x + 1)}{(x - 1)(x - 2)}$.

At $x = 1$: for $x \neq 1$, $f(x) = \frac{x + 1}{x - 2} \rightarrow \frac{2}{-1} = -2$. The limit exists, so it is a **removable** discontinuity.

At $x = 2$: the numerator tends to $3 \neq 0$ while the denominator tends to 0, so $|f| \rightarrow \infty$. It is an **infinite** discontinuity (vertical asymptote $x = 2$).

19 Make a piecewise function continuous



At $x = 0$: the left limit is $0 + 1 = 1$ and $f(0) = b$, so $b = 1$.

At $x = 3$: the left value is $3a + b = 3a + 1$ and the right limit is $10 - 3 = 7$, so $3a + 1 = 7$ and $a = 2$.

With $a = 2$, $b = 1$, the three pieces connect, and each piece is continuous, so f is continuous everywhere.

20 Cooling coffee



(a) $T(0) = 72 + 98 = 170$ degrees F. $T(10) = 72 + 98e^{-1} \approx 72 + 36.05 = 108.1$ degrees F.

(b) As $t \rightarrow +\infty$, $e^{-0.1t} \rightarrow 0$, so $T(t) \rightarrow 72$. The line $T = 72$ is a horizontal asymptote: the coffee approaches room temperature. In Celsius, $(72 - 32) \cdot \frac{5}{9} \approx 22.2$ degrees C.

(c) Solve $72 + 98e^{-0.1t} = 73$: $e^{-0.1t} = \frac{1}{98}$, so $t = 10 \ln 98 \approx 45.85$ minutes. The coffee never actually reaches 72.

21 When the IVT applies



(a) 4 lies between -3 and 5 , so yes, f must take the value 4. The value 6 is outside $[-3, 5]$, and the IVT gives no information (it may or may not be reached).

(b) $s(0) = -1 < 0$ and $s(2) = 1 > 0$, but s only takes the values -1 and 1 , so it never equals 0. The IVT requires continuity, and s has a jump at $x = 1$; the hypothesis fails.

(c) g is continuous. $g(0) = 1 > 0$ and $g(1) = \cos 1 - 1 \approx 0.540 - 1 = -0.460 < 0$. By the IVT, there is a root in $(0, 1)$.



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