

Law of Sines and Law of Cosines: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Solving an AAS triangle



Angles sum to 180° : $C = 180^\circ - 52^\circ - 71^\circ = 57^\circ$.

Law of Sines: $b = \frac{18 \sin 71^\circ}{\sin 52^\circ} \approx 21.6 \text{ cm}$ and $c = \frac{18 \sin 57^\circ}{\sin 52^\circ} \approx 19.2 \text{ cm}$.

Answer: $C = 57^\circ$, $b \approx 21.6 \text{ cm}$, $c \approx 19.2 \text{ cm}$ (the largest side, b , faces the largest angle, 71°).

2 A missing angle



$\sin B = \frac{7 \sin 48^\circ}{9} \approx 0.578$, so $B \approx 35.3^\circ$ (the obtuse choice 144.7° is impossible because $48^\circ + 144.7^\circ > 180^\circ$).

$C = 180^\circ - 48^\circ - 35.3^\circ = 96.7^\circ$ and $c = \frac{9 \sin 96.7^\circ}{\sin 48^\circ} \approx 12.0 \text{ in.}$

Answer: $B \approx 35.3^\circ$, $C \approx 96.7^\circ$, $c \approx 12.0 \text{ in.}$

3 A side from SAS



Law of Cosines: $a^2 = 12^2 + 15^2 - 2(12)(15) \cos 60^\circ = 144 + 225 - 180 = 189$.

So $a = \sqrt{189} = 3\sqrt{21} \approx 13.75$.

4 Area from two sides and the angle



$$K = \frac{1}{2}(10)(13) \sin 30^\circ = \frac{1}{2}(130)(0.5) = 32.5.$$

Answer: the sail has an area of 32.5 m^2 (about 350 ft^2).

5 Heron's formula



$$s = \frac{7 + 8 + 9}{2} = 12. \text{ Then } K = \sqrt{12 \cdot 5 \cdot 4 \cdot 3} = \sqrt{720} = 12\sqrt{5} \approx 26.83.$$

6 True or false?



- a. **True.** $\cos 90^\circ = 0$, so the term $-2ab \cos C$ vanishes.
- b. **False.** $c^2 - a^2 - b^2 = -2ab \cos C > 0$ forces $\cos C < 0$, so C is *obtuse*.
- c. **False.** Each ratio in the Law of Sines needs one known side-angle pair. With only sides known, there is none; use the Law of Cosines.

7 Which law first?



- a. AAS pattern: find $C = 67^\circ$, then use the **Law of Sines**.
- b. SAS pattern: the **Law of Cosines** gives the third side.
- c. SSS pattern: the **Law of Cosines** gives an angle, starting with the largest.
- d. SSA pattern: the **Law of Sines**, with a check for the ambiguous case (here $h = 11 \sin 25^\circ \approx 4.65 < 8 < 11$, so two triangles).

8 No triangle



$\sin B = \frac{10 \sin 40^\circ}{6} \approx 1.071 > 1$. No angle has a sine larger than 1, so **no such triangle exists**.

Equivalent check: the altitude from C is $h = 10 \sin 40^\circ \approx 6.43 > 6$, so side a is too short to reach the base.

9 Two triangles



$h = 11 \sin 35^\circ \approx 6.31 < 8 < 11$: two triangles. $\sin B = \frac{11 \sin 35^\circ}{8} \approx 0.789$, so $B_1 \approx 52.1^\circ$, $B_2 \approx 127.9^\circ$. Both satisfy $A + B < 180^\circ$.

Triangle 1: $C_1 \approx 92.9^\circ$, $c_1 = \frac{8 \sin 92.9^\circ}{\sin 35^\circ} \approx 13.9$.

Triangle 2: $C_2 \approx 17.1^\circ$, $c_2 = \frac{8 \sin 17.1^\circ}{\sin 35^\circ} \approx 4.1$.

10 A right-triangle borderline



$\sin B = \frac{10 \sin 30^\circ}{5} = 1$, so $B = 90^\circ$: exactly one triangle, a right triangle ($a = h$).

$C = 60^\circ$ and $c = \frac{5 \sin 60^\circ}{\sin 30^\circ} = 5\sqrt{3} \approx 8.66$.

11 Across the river



$\angle ATB = 180^\circ - 62^\circ - 71^\circ = 47^\circ$.

Law of Sines: $AT = \frac{80 \sin 71^\circ}{\sin 47^\circ} \approx 103.4$ ft and $BT = \frac{80 \sin 62^\circ}{\sin 47^\circ} \approx 96.6$ ft.

Answer: the tree is about 103.4 ft (31.5 m) from A and 96.6 ft (29.4 m) from B.

12 All angles from three sides



Largest angle first: $\cos C = \frac{25 + 49 - 81}{2 \cdot 5 \cdot 7} = -0.1$, so $C \approx 95.74^\circ$.

$\cos A = \frac{49 + 81 - 25}{2 \cdot 7 \cdot 9} = \frac{105}{126}$, so $A \approx 33.56^\circ$. Then

$$B = 180^\circ - 95.74^\circ - 33.56^\circ = 50.70^\circ.$$

Check: $\cos B = \frac{25 + 81 - 49}{2 \cdot 5 \cdot 9} = \frac{57}{90}$, giving 50.70° as well.

13 Area with two angles and a side



$C = 70^\circ$. Law of Sines: $a = \frac{10 \sin 50^\circ}{\sin 70^\circ} \approx 8.152$ and $b = \frac{10 \sin 60^\circ}{\sin 70^\circ} \approx 9.216$.

$$K = \frac{1}{2}ab \sin C = \frac{1}{2}(8.152)(9.216) \sin 70^\circ \approx 35.3 \text{ square units.}$$

14 Diagonals of a parallelogram



The diagonal facing the 70° angle:

$$d_1^2 = 8^2 + 11^2 - 2(8)(11) \cos 70^\circ = 185 - 176 \cos 70^\circ, \text{ so } d_1 \approx 11.17 \text{ cm.}$$

The other diagonal faces the consecutive angle 110° , where $\cos 110^\circ = -\cos 70^\circ$:

$$d_2^2 = 185 + 176 \cos 70^\circ, \text{ so } d_2 \approx 15.66 \text{ cm.}$$

15 A ship's return trip



The back-bearing at Q is $050^\circ + 180^\circ = 230^\circ$, so $\angle PQR = 230^\circ - 125^\circ = 105^\circ$.

$$PR^2 = 30^2 + 45^2 - 2(30)(45) \cos 105^\circ, \text{ so } PR \approx 60.2 \text{ mi (about 96.9 km).}$$

$$\cos P = \frac{30^2 + 60.2^2 - 45^2}{2(30)(60.2)} \approx 0.692, \text{ so } \angle QPR \approx 46.2^\circ. \text{ The bearing from P to R is}$$

$$50^\circ + 46.2^\circ = 96.2^\circ; \text{ the return bearing is } 96.2^\circ + 180^\circ = 276.2^\circ.$$

16 Spotting a fire



At A, the angle between east (090°) and 048° is 42° . At B, the direction to A is 270° , and $330^\circ - 270^\circ = 60^\circ$. So $\angle AFB = 180^\circ - 42^\circ - 60^\circ = 78^\circ$.

$$AF = \frac{5 \sin 60^\circ}{\sin 78^\circ} \approx 4.43 \text{ km}, BF = \frac{5 \sin 42^\circ}{\sin 78^\circ} \approx 3.42 \text{ km}.$$

Distance north of AB: $AF \sin 42^\circ \approx 2.96 \text{ km}$.

17 Heron, altitude and inscribed circle



a. $s = 35$, so $K = \sqrt{35 \cdot 18 \cdot 10 \cdot 7} = \sqrt{44100} = 210$.

b. From $K = \frac{1}{2} \text{ base} \times \text{height}$, the altitude is $\frac{2K}{\text{base}}$. It is smallest on the longest side:
 $h = \frac{420}{28} = 15$. (The other altitudes are $\frac{420}{17} \approx 24.7$ and $\frac{420}{25} = 16.8$.)

c. $r = \frac{210}{35} = 6$.

18 A land parcel



Triangle ABC (SAS):

$$AC^2 = 50^2 + 70^2 - 2(50)(70) \cos 60^\circ = 2500 + 4900 - 3500 = 3900, \text{ so } AC \approx 62.45 \text{ m.}$$

Its area is $\frac{1}{2}(50)(70) \sin 60^\circ \approx 1515.5 \text{ m}^2$.

Triangle ACD (SSS, Heron): $s = \frac{62.45 + 45 + 55}{2} \approx 81.22$, so

$$K \approx \sqrt{81.22(18.78)(36.22)(26.22)} \approx 1203.6 \text{ m}^2.$$

Total: about 2719 m^2 (roughly $29,300 \text{ ft}^2$).

19 Recognizing a 120 degree angle



$$\cos C = \frac{49 + 64 - 169}{2 \cdot 7 \cdot 8} = \frac{-56}{112} = -\frac{1}{2}, \text{ so } C = 120^\circ.$$

$$\text{Sine formula: } K = \frac{1}{2}(7)(8) \sin 120^\circ = 28 \cdot \frac{\sqrt{3}}{2} = 14\sqrt{3}.$$

$$\text{Heron: } s = 14, K = \sqrt{14 \cdot 7 \cdot 6 \cdot 1} = \sqrt{588} = 14\sqrt{3}. \text{ Both agree: } K \approx 24.25.$$

20 How many triangles for each side?



The altitude is $h = 12 \sin 30^\circ = 6$.

- $a < 6$: no triangle.
- $a = 6$: one triangle (right angle at B).
- $6 < a < 12$: two triangles.
- $a \geq 12$: one triangle (since $a \geq b$, angle B cannot exceed A , so the obtuse candidate is ruled out).

Sample: $a = 9$ gives $\sin B = \frac{12 \cdot 0.5}{9} = \frac{2}{3}$, so $B \approx 41.8^\circ$ or 138.2° ; both fit with 30° , confirming two triangles.

21 The circumscribed circle



$$R = \frac{13 \cdot 14 \cdot 15}{4 \cdot 84} = \frac{2730}{336} = 8.125.$$

$$\sin A = \frac{13}{2(8.125)} = \frac{13}{16.25} = 0.8, \text{ so } A \approx 53.13^\circ. \text{ Check with the Law of Cosines:}$$

$$\cos A = \frac{14^2 + 15^2 - 13^2}{2 \cdot 14 \cdot 15} = \frac{252}{420} = 0.6, \text{ and indeed } \sin A = 0.8.$$



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