

Techniques of Integration: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Integration by parts

/ 4 pts

(a) $u = \ln x$, $dv = x^2 dx$, $v = \frac{x^3}{3}$ (1 pt). $\int = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$ (1 pt).

(b) $u = x$, $dv = \sin x dx$: $[-x \cos x]_0^\pi + \int_0^\pi \cos x dx$ (1 pt) $= \pi + 0 = \pi$ (1 pt).

2 Trigonometric integrals

/ 3 pts

(a) $\cos^2 x = \frac{1+\cos 2x}{2}$ so $\int = \frac{x}{2} + \frac{\sin 2x}{4} + C$ (1 pt).

(b) $\cos^3 x = (1 - \sin^2 x) \cos x$ (1 pt); with $u = \sin x$: $\int (u^2 - u^4) du = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$ (1 pt).

3 Trigonometric substitution

/ 4 pts

(a) $x = 2 \sin \theta$, $dx = 2 \cos \theta d\theta$, $\sqrt{4 - x^2} = 2 \cos \theta$ (1 pt). The integrand becomes

$$\frac{1}{4 \sin^2 \theta} \text{ (1 pt) and } \int = -\frac{1}{4} \cot \theta + C = -\frac{\sqrt{4 - x^2}}{4x} + C \text{ (1 pt).}$$

(b) $\left[\frac{1}{3} \arctan \frac{x}{3} \right]_0^3 = \frac{1}{3} \cdot \frac{\pi}{4} = \frac{\pi}{12}$ (1 pt).

4 Partial fractions

/ 4 pts

(a) $x^2 - x - 6 = (x - 3)(x + 2)$; $A = \frac{8}{5}$, $B = -\frac{3}{5}$; so $f = \frac{8/5}{x - 3} - \frac{3/5}{x + 2}$ (2 pts).

(b) $\frac{8}{5} \ln |x - 3| - \frac{3}{5} \ln |x + 2| + C$ (1 pt).

(c) $\frac{8}{5} \ln 3 - \frac{3}{5} \ln \frac{8}{6} = \frac{8}{5} \ln 3 - \frac{3}{5} \ln \frac{4}{3} \approx 1.757780 - 0.172609 \approx 1.585$ (1 pt).

5 Improper integrals

/ 3 pts

(a) By parts: $\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x}$ (1 pt). At ∞ it tends to 0, at 1 it is -1 ; the integral equals 1 (1 pt).

(b) $\left[\frac{3}{2} x^{2/3} \right]_0^1 = \frac{3}{2}$ (1 pt).

6 Numerical integration

/ 2 pts

$h = 0.5$; values 0, 0.25, 1, 2.25, 4. $T_4 = 0.25 [0 + 2(0.25 + 1 + 2.25) + 4] = 2.75$ (1 pt). The exact value is $\frac{8}{3} \approx 2.667$, so the error is $\frac{1}{12} \approx 0.083$ (1 pt).



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