

Integrals and the Fundamental Theorem: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Checking an antiderivative



Differentiate F : $F'(x) = \frac{4x^3}{4} - 3 = x^3 - 3 = f(x)$.

Since $F' = f$, F is an antiderivative of f .

2 A general antiderivative



Use the power rule term by term: $\int 8x^3 dx = 2x^4$, $\int 6x^2 dx = 2x^3$, $\int 2x dx = x^2$, $\int 7 dx = 7x$.

So $\int f(x) dx = 2x^4 - 2x^3 + x^2 - 7x + C$.

3 A first definite integral



An antiderivative is $F(x) = x^2 + 3x$.

$F(4) = 16 + 12 = 28$ and $F(1) = 1 + 3 = 4$, so the integral equals $28 - 4 = 24$.

4 Using the properties



- a. By linearity: $2(7) + 5(-2) = 14 - 10 = 4$.
- b. Swapping the limits changes the sign: -7 .
- c. $7 - (-2) = 9$.

5 Left and right sums



Here $\Delta x = 1$. Left endpoints are 0, 1, 2: $L_3 = (0 + 2 + 4) \times 1 = 6$. Right endpoints are 1, 2, 3: $R_3 = (2 + 4 + 6) \times 1 = 12$.

The region is a triangle with base 3 and height 6, so the exact area is $\frac{1}{2} \cdot 3 \cdot 6 = 9$, which lies between 6 and 12 (it is their average).

6 Fixing the constant



Integrate: $F(x) = 3x^2 + x + C$. Then $F(2) = 12 + 2 + C = 14 + C$.

From $14 + C = 10$ we get $C = -4$, so $F(x) = 3x^2 + x - 4$.

7 True or false?



- a. True. $F(x) = \frac{x^4}{4}$ gives $F(2) - F(-2) = 4 - 4 = 0$. (The graph of x^3 is symmetric about the origin, so the signed areas cancel.)
- b. False. The left side is $\int_0^2 1 \, dx = 2$, while the right side is $2 \times 2 = 4$. An integral of a product is not the product of integrals.

8 Sine and the natural log



- a. $[-\cos x]_0^\pi = -\cos \pi + \cos 0 = 1 + 1 = 2$.
- b. $[\ln |x|]_1^e = \ln e - \ln 1 = 1 - 0 = 1$.

9 A midpoint estimate



$\Delta x = 1$; the midpoints are 0.5, 1.5, 2.5, 3.5. The heights are **1.25, 3.25, 7.25, 13.25**, which sum to 25.

So $M_4 = 25 \times 1 = 25$. Exactly: $\left[\frac{x^3}{3} + x\right]_0^4 = \frac{64}{3} + 4 = \frac{76}{3} \approx 25.33$. The error is $\frac{1}{3} \approx 0.33$.

10 Area of a tunnel opening



The parabola meets the axis at $x = \pm 3$. Area

$$= \int_{-3}^3 (9 - x^2) dx = \left[9x - \frac{x^3}{3}\right]_{-3}^3 = (27 - 9) - (-27 + 9) = 36.$$

The cross-section is 36 m^2 , about $36 \times 10.76 \approx 387 \text{ ft}^2$.

11 Filling a water tank



The net change is $\int_0^5 (12 - 2t) dt = [12t - t^2]_0^5 = 60 - 25 = 35$ gallons.

The tank then holds $80 + 35 = 115$ gallons, about $115 \times 3.785 \approx 435$ liters.

12 Two quick substitutions



a. Let $u = x^2$, $du = 2x dx$. Then $\int \cos u du = \sin u + C = \sin(x^2) + C$.

b. Let $u = 3x + 2$, $du = 3 dx$, so $dx = \frac{1}{3} du$. Then

$$\frac{1}{3} \int u^4 du = \frac{u^5}{15} + C = \frac{(3x + 2)^5}{15} + C.$$

13 Substitution with new limits

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Let $u = x^2 + 1$, so $du = 2x dx$ and $x dx = \frac{1}{2} du$. The limits become $u = 1$ and $u = 2$.

$$\frac{1}{2} \int_1^2 \frac{du}{u} = \frac{1}{2} [\ln u]_1^2 = \frac{1}{2} \ln 2 \approx 0.347.$$

14 Differentiating an integral

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By Part 1 of the Fundamental Theorem, $g'(x) = x^3 - 5$. Hence $g'(3) = 27 - 5 = 22$.

An integral from 2 to 2 is zero, so $g(2) = 0$.

15 The chain rule meets the FTC

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a. By the chain version of Part 1, $h'(x) = \sqrt{1 + (x^2)^2} \cdot 2x = 2x\sqrt{1 + x^4}$. So

$$h'(1) = 2\sqrt{2} \approx 2.83.$$

b. Swap the limits: $k(x) = -\int_5^x \sin(t^2) dt$, so $k'(x) = -\sin(x^2)$.

16 A drone changes direction

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Factor: $v(t) = (t - 2)(t - 4)$, positive on $[0, 2]$, negative on $[2, 4]$, positive on $[4, 5]$. A

position function is $s(t) = \frac{t^3}{3} - 3t^2 + 8t$.

$$s(0) = 0, s(2) = \frac{20}{3}, s(4) = \frac{16}{3}, s(5) = \frac{20}{3}.$$

Displacement: $s(5) - s(0) = \frac{20}{3} \approx 6.67$ m. Distance:

$$\frac{20}{3} + \left| \frac{16}{3} - \frac{20}{3} \right| + \left(\frac{20}{3} - \frac{16}{3} \right) = \frac{28}{3} \approx 9.33 \text{ m.}$$

17 Sine cubed times cosine

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Let $u = \sin x$, $du = \cos x \, dx$. When $x = 0$, $u = 0$; when $x = \pi/2$, $u = 1$.

$$\int_0^1 u^3 \, du = \left[\frac{u^4}{4} \right]_0^1 = \frac{1}{4}.$$

18 Logarithm substitution

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Let $u = \ln x$, $du = \frac{dx}{x}$. The limits become $\ln 1 = 0$ and $\ln e^2 = 2$.

$$\int_0^2 u \, du = \left[\frac{u^2}{2} \right]_0^2 = 2.$$

19 Absolute value integral

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$|x - 1| = 1 - x$ on $[-1, 1]$ and $x - 1$ on $[1, 3]$. By additivity,

$$\int_{-1}^1 (1 - x) \, dx = \left[x - \frac{x^2}{2} \right]_{-1}^1 = \frac{1}{2} - \left(-\frac{3}{2} \right) = 2 \text{ and}$$
$$\int_1^3 (x - 1) \, dx = \left[\frac{x^2}{2} - x \right]_1^3 = \frac{3}{2} - \left(-\frac{1}{2} \right) = 2.$$

The total is 4. (Each piece is a triangle with base 2 and height 2.)

20 Marginal cost

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The increase is $C(200) - C(100) = \int_{100}^{200} (0.06q + 4) \, dq = [0.03q^2 + 4q]_{100}^{200}$.

$= (1,200 + 800) - (300 + 400) = 2,000 - 700 = 1,300$. The cost rises by \$1,300.

21 From Riemann sums to an integral



With $\Delta x = \frac{1}{n}$ and $x_i = \frac{i}{n}$:

$$R_n = \sum_{i=1}^n \frac{i^2}{n^2} \cdot \frac{1}{n} = \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6n^2}.$$

For $n = 10$: $R_{10} = \frac{11 \cdot 21}{600} = 0.385$.

As $n \rightarrow \infty$, $\frac{(n+1)(2n+1)}{6n^2} \rightarrow \frac{2}{6} = \frac{1}{3}$, which equals $\int_0^1 x^2 dx$, as the Fundamental Theorem predicts.

22 A radical integrand



Let $u = x^2 + 5$, $x dx = \frac{1}{2} du$; the limits become 5 and 9.

$$\frac{1}{2} \int_5^9 u^{1/2} du = \frac{1}{2} \cdot \frac{2}{3} [u^{3/2}]_5^9 = \frac{1}{3} (27 - 5\sqrt{5}).$$

Numerically, $\frac{1}{3} (27 - 11.180) \approx 5.273$.



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