

Integrals and the Fundamental Theorem: math practice, College – download the PDF

22 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

1 Checking an antiderivative

★ ★ ★

Show that $F(x) = \frac{x^4}{4} - 3x$ is an antiderivative of $f(x) = x^3 - 3$.

2 A general antiderivative

★ ★ ★

Find the general antiderivative of $f(x) = 8x^3 - 6x^2 + 2x - 7$.

3 A first definite integral

★ ★ ★

Evaluate $\int_1^4 (2x + 3) dx$.

4 Using the properties



Suppose $\int_1^5 f(x) dx = 7$ and $\int_1^5 g(x) dx = -2$. Find:

- a. $\int_1^5 (2f(x) + 5g(x)) dx$
- b. $\int_5^1 f(x) dx$
- c. $\int_1^5 (f(x) - g(x)) dx$

5 Left and right sums



For $f(x) = 2x$ on $[0, 3]$ with $n = 3$ strips, compute the left sum and the right sum. Then compare them with the exact area under the line.

6 Fixing the constant



Find the function F with $F'(x) = 6x + 1$ and $F(2) = 10$.

7 True or false?



Decide whether each statement is true or false, and justify.

- a. $\int_{-2}^2 x^3 dx = 0$.
- b. $\int_0^2 (1 \cdot 1) dx = \left(\int_0^2 1 dx \right) \left(\int_0^2 1 dx \right)$.

8 Sine and the natural log



Evaluate:

a. $\int_0^{\pi} \sin x \, dx$

b. $\int_1^e \frac{1}{x} \, dx$

9 A midpoint estimate

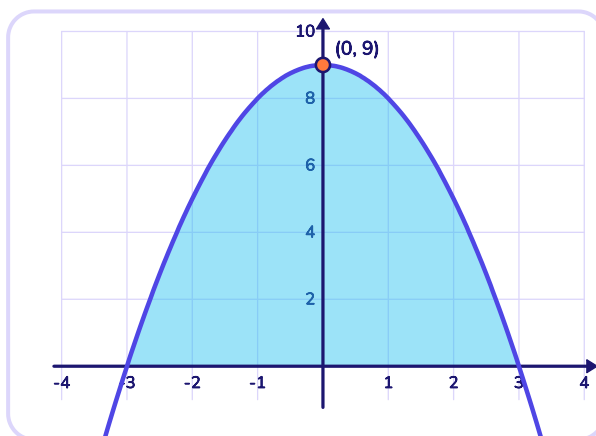


Use the midpoint rule with $n = 4$ strips to approximate $\int_0^4 (x^2 + 1) \, dx$. Then compute the exact value and the error.

10 Area of a tunnel opening



The cross-section of a tunnel is the region between the x -axis and the parabola $y = 9 - x^2$, with x and y in meters.



Find its area in square meters, and convert it to square feet ($1 \text{ m}^2 \approx 10.76 \text{ ft}^2$).

11 Filling a water tank

A large tank contains 80 gallons of water at time $t = 0$. Water is pumped in at a rate of $r(t) = 12 - 2t$ gallons per minute for $0 \leq t \leq 5$ minutes. How many gallons does the tank contain after 5 minutes? Give the answer in liters too ($1 \text{ gal} \approx 3.785 \text{ L}$).

12 Two quick substitutions

Find each indefinite integral.

a. $\int 2x \cos(x^2) dx$

b. $\int (3x + 2)^4 dx$

13 Substitution with new limits

Evaluate $\int_0^1 \frac{x}{x^2 + 1} dx$.

14 Differentiating an integral

Let $g(x) = \int_2^x (t^3 - 5) dt$. Find $g'(x)$, then $g'(3)$ and $g(2)$.

15 The chain rule meets the FTC

Find the derivatives:

a. $h(x) = \int_0^{x^2} \sqrt{1 + t^2} dt$; also compute $h'(1)$.

b. $k(x) = \int_x^5 \sin(t^2) dt$.

16 A drone changes direction

★★★

A drone flies along a straight line with velocity $v(t) = t^2 - 6t + 8$ m/s for $0 \leq t \leq 5$ seconds. Find its displacement and the total distance traveled.

17 Sine cubed times cosine

★★★

Evaluate $\int_0^{\pi/2} \sin^3 x \cos x \, dx$.

18 Logarithm substitution

★★★

Evaluate $\int_1^{e^2} \frac{\ln x}{x} \, dx$.

19 Absolute value integral

★★★

Compute $\int_{-1}^3 |x - 1| \, dx$. (Hint: split the interval where $x - 1$ changes sign.)

20 Marginal cost

★★★

A factory's marginal cost is $C'(q) = 0.06q + 4$ dollars per unit when q units have been made. By how many dollars does the total cost rise when production goes from 100 to 200 units?

21 From Riemann sums to an integral

★★★

Let R_n be the right-endpoint sum of $f(x) = x^2$ on $[0, 1]$ with n strips. Use

$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ to find a formula for R_n , compute R_{10} , and find

$\lim_{n \rightarrow \infty} R_n$.

22

A radical integrand

★★★

Evaluate $\int_0^2 x\sqrt{x^2+5} \, dx$ exactly, then give a decimal approximation.

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