

Solving Inequalities: math lesson, Grade 7 – download the PDF

A theme park ride says, “You must be at least 48 inches tall.” A phone plan says, “No more than 10 GB of fast data.” Neither sentence names a single number. Each one describes a whole **range** of allowed values. In this chapter you will learn to write these rules with symbols, solve them like equations (with one important twist), and show the answers on a number line.

1. What is an inequality?

An **equation** says two quantities are equal, like $x + 3 = 10$. It has one solution: $x = 7$. An **inequality** says that one quantity is bigger or smaller than another. For example, $x + 3 > 10$ is true when x is 8, 9, 12.5, or 1,000. It is also false for $x = 7$ or $x = 2$. So an inequality usually has *infinitely many* solutions, and we describe them all at once.

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Definition: inequality

An inequality is a mathematical sentence that compares two expressions using one of the symbols $<$, $>$, $\leq$, or $\geq$. A **solution** is any number that makes the sentence true when it replaces the variable. The **solution set** is the collection of all solutions.

2. Reading the inequality symbols

The symbol always “points” to the smaller value, and its wide, open side faces the bigger value. The two symbols with a line underneath also allow equality.

Symbol	Read as	Everyday phrases
$<$	is less than	under, below, fewer than
$>$	is greater than	over, above, more than, exceeds
$\leq$	is less than or equal to	at most, no more than, maximum, up to
$\geq$	is greater than or equal to	at least, no less than, minimum, or more



Watch the phrases

“At most 10” means ≤ 10 (10 is allowed and nothing above it). “At least 10” means ≥ 10 . People mix these up all the time, so read slowly. Also remember that $x > 3$ and $3 < x$ say exactly the same thing.

3. Solutions and graphs on a number line

To check whether a number is a solution, substitute it and see whether the sentence is true. We then **graph** the whole solution set on a number line using two ideas.

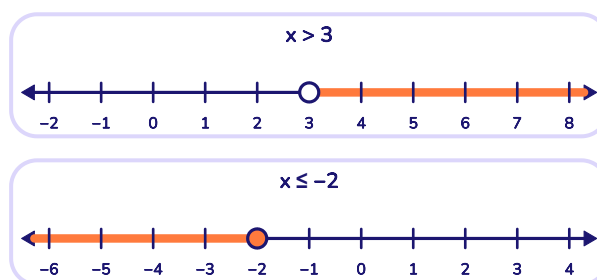
- A circle marks the **boundary number**. An **open circle** means the boundary is *not* included ($<$ or $>$). A **closed circle** means it *is* included ($\leq$ or $\geq$).
- The **shaded ray** goes toward all the values that work: to the right for “greater,” to the left for “less.”



Zyro's tip

On my home planet we say an open circle is an empty seat: the boundary number is not allowed to sit down. A closed circle is a filled seat, so that number is part of the solution!

Here are two graphs. The first shows all numbers greater than 3 (but not 3 itself). The second shows all numbers less than or equal to -2 .



Example 1: testing values

Is 6 a solution of $2x - 1 > 9$? Substitute: $2(6) - 1 = 11$, and $11 > 9$ is true, so yes. Is 5 a solution? $2(5) - 1 = 9$, and $9 > 9$ is false, so no. Because 5 is the boundary and the symbol is strict, 5 is not included.

4. Solving by adding, subtracting, multiplying, or dividing

Solving an inequality means isolating the variable, just as with an equation. Think of a balance scale that is tilted: if you add the same weight to both pans, the same side stays heavier.

P Properties (positive numbers)

For any number c : adding or subtracting c on both sides keeps the inequality sign the same. Multiplying or dividing both sides by a **positive** number also keeps the sign the same.

E Example 2: subtract, then check

Solve $x - 7 \geq 2$. Add 7 to both sides: $x \geq 9$. Check with a number from the solution set, $x = 10$: $10 - 7 = 3$, and $3 \geq 2$ is true. Check a number outside it, $x = 8$: $1 \geq 2$ is false. The solution set is every number from 9 upward, with 9 included.

I 5. The big twist: negative numbers flip the sign

Start with the true statement $2 < 5$. Multiply both sides by -3 : you get -6 and -15 . On the number line -6 is to the *right* of -15 , so $-6 > -15$. The order reversed!

Start	Multiply by -3	True statement
$2 < 5$	-6 and -15	$-6 > -15$
$-4 < 1$	12 and -3	$12 > -3$
$7 > 0$	-21 and 0	$-21 < 0$

P**Property (negative numbers)**

When you multiply or divide both sides of an inequality by a **negative** number, you must **reverse the inequality sign**: $<$ becomes $>$, $\leq$ becomes $\geq$, and vice versa.

E**Example 3: dividing by a negative**

Solve $-4x \geq 20$. Divide both sides by -4 and flip the sign: $x \leq -5$. Check $x = -6$: $-4(-6) = 24$, and $24 \geq 20$ is true. Check $x = 0$: $0 \geq 20$ is false. The graph is a closed circle at -5 with the ray shaded to the left.

6. Solving $px + q > r$ step by step

Most inequalities in this chapter look like $px + q > r$ (or with $<$, $\leq$, $\geq$). You undo the operations in the reverse order, exactly as in a two-step equation.

M**Method**

1. Undo the addition or subtraction: subtract q from both sides.
2. Undo the multiplication: divide both sides by p . **If p is negative, flip the sign.**
3. Write the solution set and graph it (open or closed circle, then shade).
4. Check one number from the shaded part in the original inequality.

E Example 4: positive coefficient

Solve $5x - 8 > 22$. Add 8: $5x > 30$. Divide by 5 (positive, so no flip): $x > 6$. Check $x = 7$: $35 - 8 = 27 > 22$. True.

E Example 5: negative coefficient

Solve $9 - 2x \geq 3$. Subtract 9: $-2x \geq -6$. Divide by -2 and flip: $x \leq 3$. Check $x = 0$: $9 \geq 3$ is true. Check $x = 5$: $9 - 10 = -1 \geq 3$ is false. Good.

Flip only when you multiply or divide by a negative

In $x - 6 > -2$ there is a negative number, but you only *add* 6, so the sign stays: $x > 4$. Do not flip when you add or subtract, even if negatives appear.

7. Writing inequalities from word problems

Real situations are full of limits. Translate them in four moves: name the variable, find the key phrase, write the inequality, solve it.

Words in the problem	Symbol
costs less than, must be under, fewer than	$<$
more than, exceeds, over	$>$
no more than, at most, cannot exceed, up to	$\leq$
no less than, at least, needs a minimum of	$\geq$

E**Example 6: bowling budget**

Renting shoes costs 4 dollars, and each game costs 3.50 dollars. Jordan can spend at most 25 dollars. Let g be the number of games. The total cost is $4 + 3.5g$, and it must be at most 25: $4 + 3.5g \leq 25$. Subtract 4: $3.5g \leq 21$. Divide by 3.5: $g \leq 6$. Jordan can bowl 6 games or fewer.

8. Interpreting the solution set in context

The algebra gives numbers; the story decides which of them make sense. A game cannot be played 2.4 times, and a negative number of tickets does not exist. Always ask: **Which values are possible in this situation?**

E**Example 7: reading the answer**

Ben has 62 points and wants at least 90. Each level is worth 7 points. With n more levels, $62 + 7n \geq 90$, so $7n \geq 28$ and $n \geq 4$. The solutions are all numbers from 4 up, but levels are whole numbers, so Ben needs 4, 5, 6, ... more levels. The smallest choice is 4 levels, which gives exactly 90 points.

**Rounding goes in the direction of the rule**

If a solution is $n \leq 6.4$ buses, you can use 6 buses, not 7. If it is $n \geq 9.6$ weeks, you need 10 weeks, not 9. Always test the whole number you pick in the original problem.

Key takeaways

- The symbols $<$, $>$, $\leq$, $\geq$ compare values; “at most” is $\leq$ and “at least” is $\geq$.
- An inequality usually has infinitely many solutions, shown on a number line: open circle for $<$ or $>$, closed circle for $\leq$ or $\geq$, then shade.
- Adding, subtracting, or multiplying and dividing by a positive number keeps the sign.
- Multiplying or dividing by a negative number **reverses** the sign.
- To solve $px + q > r$: subtract q , divide by p , flip if $p < 0$, then check one value.
- In word problems, define the variable, translate the key phrase, solve, and interpret the answer for the real situation.



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