

Points, Lines, Planes, and Angle Relationships: math lesson, Grade 10 – download the PDF

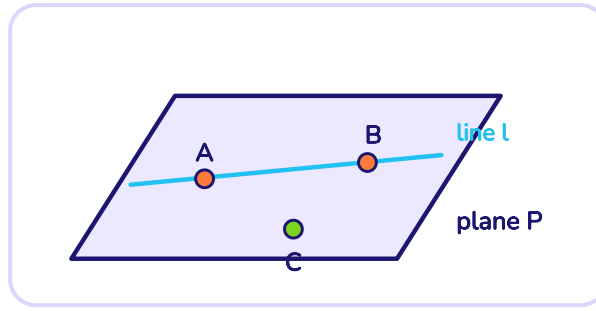
Every proof, blueprint, and video-game map starts from a few simple ideas: points, lines, planes, and the angles they create. In this chapter you will learn the basic vocabulary of geometry, then use it to measure segments, find midpoints, split angles, and discover which angle pairs are always equal. You will also draw exact figures with nothing but a compass and a straightedge.

1. Undefined terms: point, line, plane

Geometry has to start somewhere. A few words are accepted without a formal definition, and all other terms are built from them. These are the **undefined terms**.

D Point, line, plane

- A **point** names a location. It has no size. We name it with a capital letter, such as **A**.
- A **line** is a straight path that extends forever in both directions and has no thickness. It is named by two of its points, $\overleftrightarrow{AB}$, or by a lowercase letter such as ***l***.
- A **plane** is a flat surface that extends forever in all directions and has no thickness. It is named by a capital letter such as ***P***, or by three points that are not on one line.



Points that lie on the same line are **collinear**. Points that lie in the same plane are **coplanar**. Two points always determine exactly one line, and three points that are not collinear determine exactly one plane. Think of a camera tripod: three legs always touch the floor in a single flat plane, which is why it never wobbles.

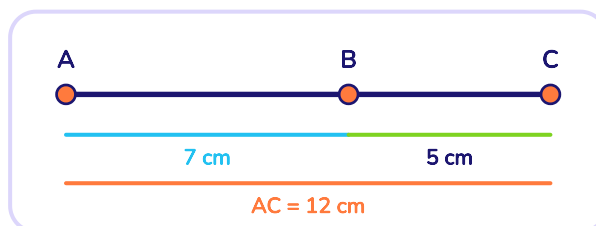
2. Segments, rays, and the segment addition postulate

D Segment and ray

A **segment** $\overline{AB}$ is the part of a line between two endpoints A and B , including both. Its length is written AB . A **ray** $\overrightarrow{AB}$ starts at the endpoint A and passes through B , then goes on forever. Two segments with equal lengths are **congruent**.

P Segment Addition Postulate

If B is between A and C on a line, then $AB + BC = AC$.



E**Example 1: using the postulate**

Point B is between A and C , with $AB = 7 \text{ cm}$ and $BC = 5 \text{ cm}$. Then

$AC = AB + BC = 7 + 5 = 12 \text{ cm}$. If instead you know $AC = 12$ and $AB = 7$, subtract: $BC = 12 - 7 = 5 \text{ cm}$.

**Watch out**

The postulate only works when B is really between A and C on the same line. If the three points form a triangle, $AB + BC$ is larger than AC .

3. Midpoint and distance formulas**D****Midpoint**

The **midpoint** of $\overline{AB}$ is the point M of the segment that splits it into two congruent segments: $AM = MB$. A line, ray, or segment through M that cuts $\overline{AB}$ there is called a **segment bisector**.

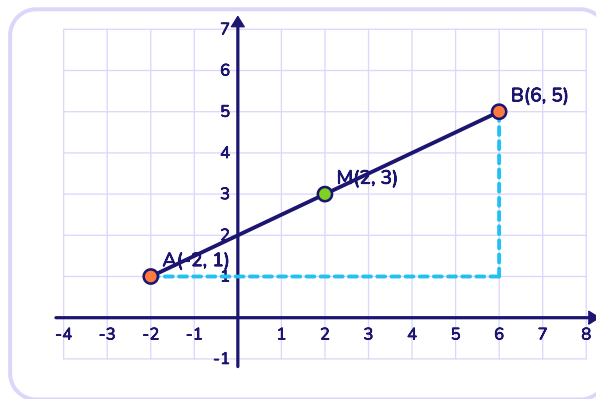
P

Midpoint and distance formulas

For $A(x_1, y_1)$ and $B(x_2, y_2)$ in the coordinate plane:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The midpoint formula averages the coordinates. The distance formula is the Pythagorean theorem applied to the horizontal and vertical legs of a right triangle.



E

Example 2: midpoint and length

Let $A(-2, 1)$ and $B(6, 5)$. Midpoint: $M = \left(\frac{-2 + 6}{2}, \frac{1 + 5}{2} \right) = (2, 3)$. Length: the horizontal leg is $6 - (-2) = 8$ and the vertical leg is $5 - 1 = 4$, so $AB = \sqrt{8^2 + 4^2} = \sqrt{80} = 4\sqrt{5} \approx 8.94$ units.

M**Method: find a missing endpoint**

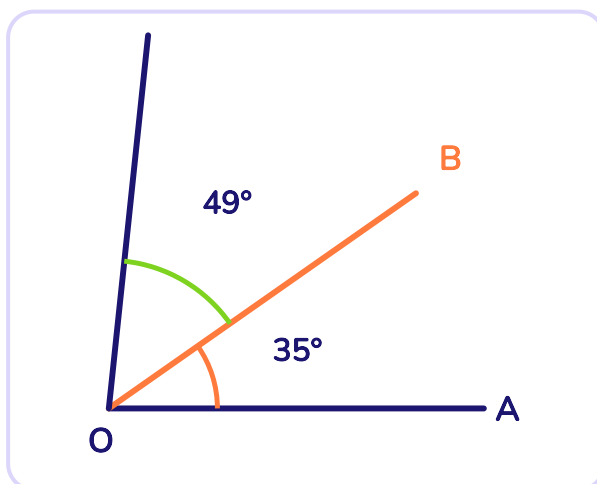
1. Write the midpoint formula with the unknown coordinates.
2. Set each average equal to the matching coordinate of M .
3. Solve: $x_2 = 2x_M - x_1$ and $y_2 = 2y_M - y_1$.

4. The angle addition postulate and angle bisectors**D****Angle**

An **angle** is formed by two rays with a common endpoint, the **vertex**. We write $\angle AOC$, with the vertex in the middle, and its measure $m\angle AOC$ in degrees. An angle is **acute** if its measure is less than 90° , **right** if it equals 90° , **obtuse** if it is between 90° and 180° , and **straight** if it equals 180° .

P**Angle Addition Postulate**

If point B lies in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.

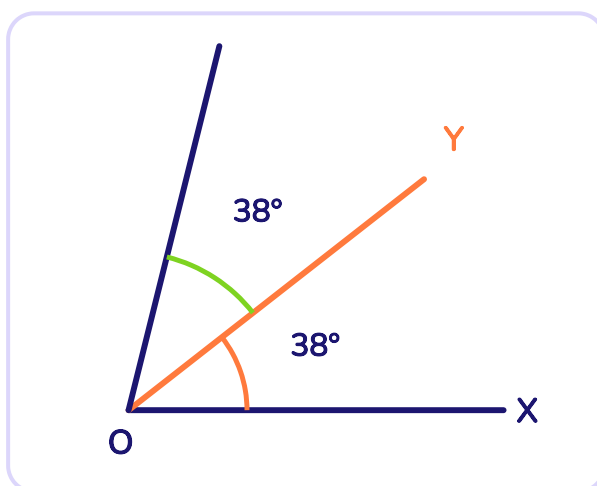


E Example 3: adding angles

In the figure, $m\angle AOB = 35^\circ$ and $m\angle BOC = 49^\circ$, so $m\angle AOC = 35^\circ + 49^\circ = 84^\circ$.

D Angle bisector

An **angle bisector** is a ray inside an angle that splits it into two congruent angles. If ray OY bisects $\angle XOZ$, then $m\angle XOY = m\angle YOZ = \frac{1}{2} m\angle XOZ$.



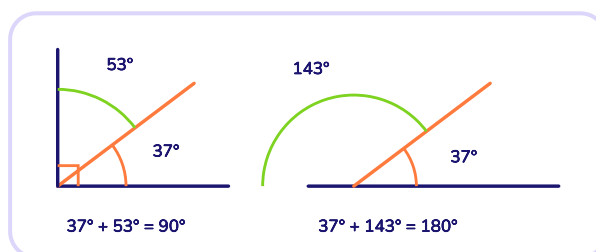
For an angle of 76° , the bisector creates two angles of 38° . In algebra problems, set the two expressions equal to each other, solve for the variable, and substitute back.

5. Complementary and supplementary angles

D Complementary and supplementary

- Two angles are **complementary** if their measures add up to 90° .
- Two angles are **supplementary** if their measures add up to 180° .

The complement of an angle of measure x is $90^\circ - x$, and its supplement is $180^\circ - x$. The angles do not need to touch each other.



! Zyro's tip

On my home planet we remember it with the alphabet: C comes before S, and 90 comes before 180. Complementary goes with 90, supplementary goes with 180.

6. Linear pairs and vertical angles

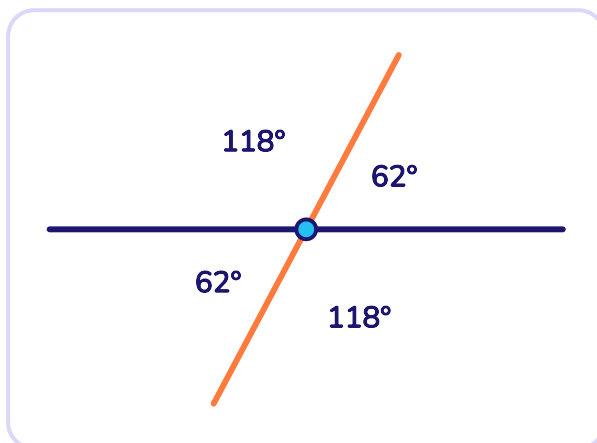
D Linear pair and vertical angles

- A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays. Together they form a straight line.
- **Vertical angles** are the two angles that are opposite each other when two lines intersect. They share only the vertex.



Linear Pair Postulate and Vertical Angles Theorem

The angles of a linear pair are supplementary: their measures add up to 180° . Vertical angles are congruent.



Why are vertical angles equal? Call them $\angle 1$ and $\angle 3$, and let $\angle 2$ be between them. Then $m\angle 1 + m\angle 2 = 180^\circ$ and $m\angle 2 + m\angle 3 = 180^\circ$. Subtracting $m\angle 2$ from both equations gives $m\angle 1 = m\angle 3$.



Example 4: all four angles from one

Two lines intersect and one angle measures 62° . Its vertical angle also measures 62° . Each neighbor forms a linear pair with it, so each measures $180^\circ - 62^\circ = 118^\circ$.
Check: $62 + 118 + 62 + 118 = 360$.



Common mistake

Every linear pair is supplementary, but not every supplementary pair is a linear pair. Two angles of 100° and 80° in different places are supplementary without sharing a side.

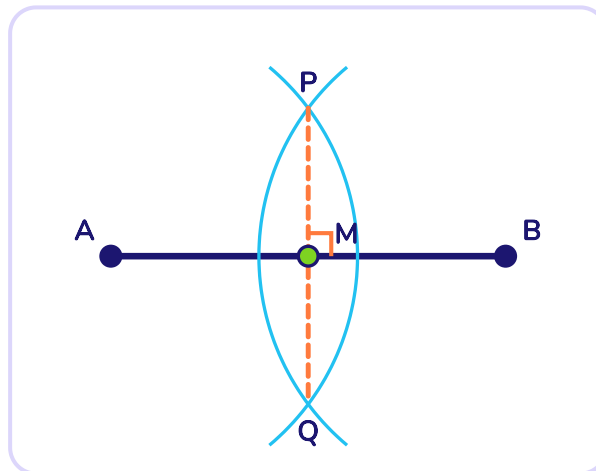
7. Constructions with compass and straightedge

A **construction** uses only a compass (to draw arcs and copy lengths) and an unmarked straightedge (to draw straight lines). No ruler measurements and no protractor are allowed. The arcs you leave on the paper are the proof that the figure is exact.

M

Method: perpendicular bisector of $\overline{AB}$

1. Open the compass to more than half of AB .
2. Put the point on A and draw one arc above and one below the segment.
3. Keep the same opening, put the point on B , and draw two more arcs that cross the first ones at P and Q .
4. Draw line PQ . It crosses $\overline{AB}$ at its midpoint M and is perpendicular to it.





Method: bisect an angle $\angle ABC$

1. Draw an arc centered at B that crosses both sides, at P and Q .
2. Using one common opening that is more than half of PQ , draw an arc centered at P and another centered at Q . They meet at R inside the angle.
3. Draw ray BR . It is the angle bisector.

It works because triangles BPR and BQR have three pairs of equal sides, so they are congruent and the angles at B are equal.

Key takeaways

- Point, line, and plane are undefined terms. Two points determine a line; three non-collinear points determine a plane.
- Segment Addition Postulate: if B is between A and C , then $AB + BC = AC$.
- Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. Distance: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- Angle Addition Postulate: $m\angle AOB + m\angle BOC = m\angle AOC$. A bisector gives two equal halves.
- Complementary: sum 90° . Supplementary: sum 180° . Linear pairs are supplementary. Vertical angles are congruent.
- Constructions use only a compass and a straightedge; equal compass openings give congruent triangles.



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