

Geometric Probability and Conditional Probability: math lesson, Grade 10 – download the PDF

How likely is it that a dart lands in the bull's-eye? That a student who plays a sport also has a job? That the three fastest runners finish in a particular order? In this chapter you will measure chances with lengths and areas, read two-way tables, update a probability when new information arrives, and count arrangements with permutations and combinations.

1. Sample spaces and events

D Definitions

An **experiment** is a process with an uncertain result. Its **sample space** S is the set of all possible outcomes. An **event** is any subset of S . When all outcomes are equally likely,

$$P(A) = \frac{\text{number of outcomes in } A}{\text{number of outcomes in } S}, \quad 0 \leq P(A) \leq 1.$$

A probability of **0** means the event is impossible, and a probability of **1** means it is certain. To avoid missing outcomes, list them in an organized way: a table, a tree, or the **Fundamental Counting Principle**, which says that if one choice can be made in m ways and a second choice in n ways, the two choices together can be made in $m \cdot n$ ways.

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Example 1: coin and die

You flip a coin and roll a die. The sample space has $2 \cdot 6 = 12$ outcomes. The event “heads and an even number” contains $(H, 2), (H, 4), (H, 6)$, so $P = \frac{3}{12} = \frac{1}{4}$.

2. Geometric probability: length and area

When an outcome is a *point* chosen at random in a region, you cannot list the outcomes, but you can compare sizes. Assuming every point is equally likely,

$$P(\text{event}) = \frac{\text{measure of the favorable region}}{\text{measure of the whole region}},$$

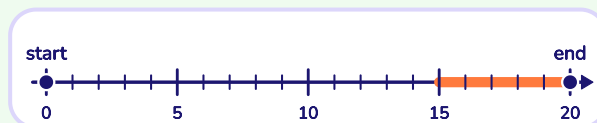
where “measure” means length (for a segment or a time interval) or area (for a plane region).

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Example 2: waiting for a bus

A bus comes every 20 minutes and you arrive at a random moment. The chance of waiting more than 15 minutes is the length of the orange part divided by the total

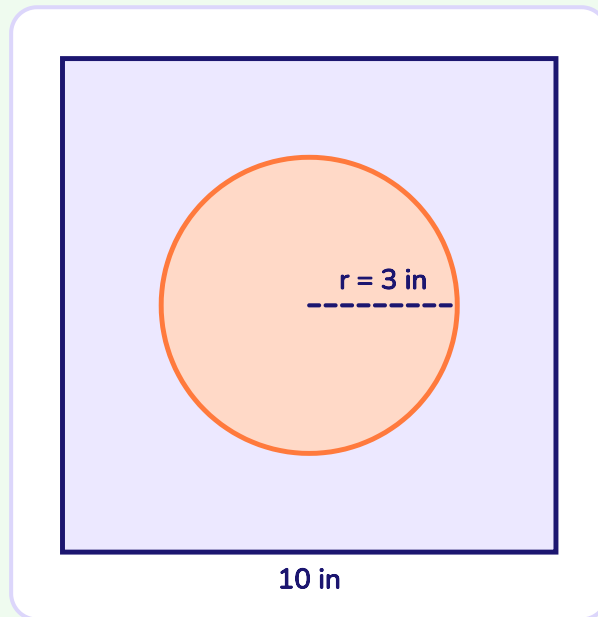
length: $\frac{20 - 15}{20} = \frac{1}{4}$.



E**Example 3: a dartboard**

A square board is 10 in wide and has a centered circle of radius 3 in. A dart lands at a random point of the board. The circle's area is $\pi \cdot 3^2 = 9\pi$ and the square's area is $10^2 = 100$, so

$$P(\text{circle}) = \frac{9\pi}{100} \approx 0.283.$$

**Common mistake**

Compare areas with areas and lengths with lengths, in the same unit. Never divide a radius by a side length: the probability uses πr^2 , not r .

3. Unions, intersections, and complements

D Vocabulary

The **complement** A^c is “not A ”. The **intersection** $A \cap B$ is “ A and B ”. The **union** $A \cup B$ is “ A or B (or both)”.

P Rules

$$P(A^c) = 1 - P(A), \quad P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If A and B cannot happen together (they are **mutually exclusive**), then $P(A \cap B) = 0$ and the union rule becomes $P(A \cup B) = P(A) + P(B)$.

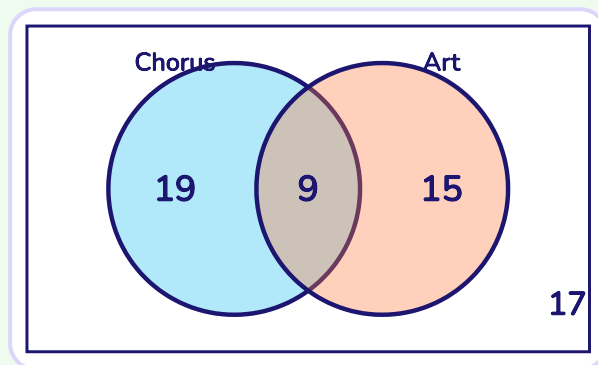
We subtract $P(A \cap B)$ because outcomes in both events were counted twice. A Venn diagram makes this visible.

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Example 4: school clubs

Among 60 students, 28 are in chorus, 24 are in art, and 9 are in both. Then $28 - 9 = 19$ are in chorus only, $24 - 9 = 15$ in art only, and $19 + 9 + 15 = 43$ are in at least one club, so 17 are in neither.

$$P(\text{chorus or art}) = \frac{28 + 24 - 9}{60} = \frac{43}{60}, \quad P(\text{neither}) = 1 - \frac{43}{60} = \frac{17}{60}.$$



4. Independent events

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Independent events

Two events are **independent** when the occurrence of one does not change the probability of the other. In that case

$$P(A \cap B) = P(A) \cdot P(B).$$

Flipping a coin twice gives independent events: the second flip does not remember the first. Drawing two cards from a deck without putting the first back gives *dependent* events, because the first draw changes what is left.

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Method: testing independence

1. Compute $P(A)$, $P(B)$ and $P(A \cap B)$.
2. Compute $P(A) \cdot P(B)$.
3. If the two numbers are equal, the events are independent; otherwise they are dependent.

5. Two-way frequency tables

A **two-way frequency table** sorts the same group by two categories at once. The totals on the margins give the **marginal** counts, and the inner cells give the **joint** counts. Here are 100 students sorted by sport and part-time job.

	Has a job	No job	Total
Plays a sport	24	36	60
No sport	10	30	40
Total	34	66	100

The joint probability that a student plays a sport *and* has a job is $\frac{24}{100}$. The marginal probability of having a job is $\frac{34}{100}$. Always read the question carefully: “of the students who play a sport” tells you to restrict the table to one row.

6. Conditional probability

D Conditional probability

The probability of A given that B has occurred, with $P(B) > 0$, is

$$P(A | B) = \frac{P(A \cap B)}{P(B)}.$$

Equivalently, $P(A \cap B) = P(B) \cdot P(A | B)$.

Conditioning shrinks the sample space to B . In a table, you divide by the total of the given row or column, not by the grand total.

E Example 5: using the table

Given that a student plays a sport, the chance of having a job is

$$P(\text{job} | \text{sport}) = \frac{24}{60} = \frac{2}{5} = 0.4.$$

Since $P(\text{job}) = 0.34 \neq 0.4$, the events “plays a sport” and “has a job” are *not* independent. Notice also that A and B are independent exactly when $P(A | B) = P(A)$.

Common mistake

$P(A | B)$ and $P(B | A)$ are usually different. Here $P(\text{sport} | \text{job}) = \frac{24}{34}$, not $\frac{24}{60}$.



Zyro's tip

On my planet we say: “given” means “look only inside that group.” Cross out everything outside the group before you count!

7. Permutations and combinations

To count outcomes, ask whether *order matters*. A **permutation** is an ordered arrangement; a **combination** is an unordered selection. With $n! = n(n-1) \cdots 2 \cdot 1$,

$${}_nP_r = \frac{n!}{(n-r)!}, \quad {}_nC_r = \frac{n!}{r!(n-r)!}.$$



Example 6: podium or team?

Eight runners race. The number of ways to award gold, silver and bronze (order matters) is ${}_8P_3 = 8 \cdot 7 \cdot 6 = 336$. The number of ways to choose 3 runners to represent the school (order does not matter) is ${}_8C_3 = \frac{336}{3!} = 56$.



Method: probability with counting

1. Count the total number of equally likely outcomes.
2. Count the favorable outcomes with the same rule (same order convention).
3. Divide favorable by total.

For instance, if 3 students are chosen from 5 juniors and 4 seniors, the probability that all are juniors is $\frac{{}_5C_3}{{}_9C_3} = \frac{10}{84} = \frac{5}{42}$.

Key takeaways

- $P(A) = \frac{\text{favorable}}{\text{total}}$ for equally likely outcomes, and $P(A^c) = 1 - P(A)$.
- Geometric probability compares lengths with lengths or areas with areas.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
- Independent events satisfy $P(A \cap B) = P(A)P(B)$, or equivalently $P(A | B) = P(A)$.
- $P(A | B) = \frac{P(A \cap B)}{P(B)}$: divide by the given group.
- Order matters: permutations ${}_nP_r$. Order does not matter: combinations ${}_nC_r$.



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