

Polynomial and Rational Functions Review: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Degree and end behavior



The highest power is x^5 , so the degree is 5 and the leading coefficient is -2 . The y -intercept is $f(0) = -7$.

The degree is odd and the leading coefficient is negative, so the graph rises on the left ($f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$) and falls on the right ($f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$).

2 Four end behaviors



- Odd degree, positive leading coefficient: falls on the left, rises on the right.
- Even degree, negative leading coefficient: falls on both ends.
- Odd degree, negative leading coefficient: rises on the left, falls on the right.
- Even degree, positive leading coefficient: rises on both ends.

3 Zeros and multiplicities



Set each factor to zero: $x = 0$ (multiplicity 2), $x = 4$ (multiplicity 1), $x = -1$ (multiplicity 3).

At 0 the multiplicity is even, so the graph touches the axis and turns around. At 4 it crosses. At -1 the multiplicity is odd, so it crosses while flattening out. The degree is $2 + 1 + 3 = 6$.

4 A first synthetic division



Use $c = 2$ and coefficients $1, -4, 1, 6$.

$c = 2$	1	-4	1	6
Multiply by c		2	-4	-6
Add	1	-2	-3	0

The quotient is $x^2 - 2x - 3$ and the remainder is 0 . Therefore

$$x^3 - 4x^2 + x + 6 = (x - 2)(x^2 - 2x - 3) = (x - 2)(x - 3)(x + 1).$$

5 Remainders by evaluating



By the Remainder Theorem the remainder is $P(c)$ for the divisor $x - c$.

a. $P(3) = 27 - 15 + 4 = 16$.

b. The divisor is $x - (-1)$, so the remainder is $P(-1) = -1 + 5 + 4 = 8$.

6 Simple asymptotes



The denominator is zero at $x = -4$ and the numerator is not, so the vertical asymptote is $x = -4$. The degrees are equal (both 1), so the horizontal asymptote is the ratio of the leading coefficients: $y = \frac{3}{1} = 3$.

7 A quick inequality



The critical numbers are -2 and 5 . The parabola opens upward, so the product is negative only between its zeros. Check $x = 0$: $(-5)(2) = -10 < 0$. The solution is $(-2, 5)$.

8 True or false?



- a. True. For example $x^4 + 1 > 0$ for every real x .
- b. True. Non-real zeros come in conjugate pairs, so a cubic has 3 zeros with an even number of non-real ones (0 or 2). At least one zero is real.
- c. False. Since $f(x) = \frac{(x-1)(x+1)}{x-1} = x+1$ for $x \neq 1$, the factor cancels. There is a hole at $(1, 2)$, not an asymptote.

9 Factor Theorem



$P(-3) = -27 + 18 + 15 - 6 = 0$, so $x + 3$ is a factor by the Factor Theorem. Divide with $c = -3$:

$c = -3$	1	2	-5	-6
Multiply by c		-3	3	6
Add	1	-1	-2	0

The quotient is $x^2 - x - 2 = (x - 2)(x + 1)$. So $P(x) = (x + 3)(x - 2)(x + 1)$.

10 Finding a missing coefficient



The Factor Theorem gives $P(2) = 8 + 4k - 6 - 10 = 4k - 8 = 0$, so $k = 2$ and $P(x) = x^3 + 2x^2 - 3x - 10$.

$c = 2$	1	2	-3	-10
Multiply by c		2	8	10
Add	1	4	5	0

The quotient is $x^2 + 4x + 5$. Its discriminant is $16 - 20 = -4 < 0$, so

$$x = \frac{-4 \pm 2i}{2} = -2 \pm i. \text{ The zeros are } 2, -2 + i, \text{ and } -2 - i.$$

11 Rational Root Theorem at work



Divisors of the constant term 6: $\pm 1, \pm 2, \pm 3, \pm 6$. Divisors of the leading coefficient 2: $1, 2$.

Candidates: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$.

Test $x = 2$: $16 + 4 - 26 + 6 = 0$. Divide:

$c = 2$	2	1	-13	6
Multiply by c		4	10	-6
Add	2	5	-3	0

The quotient is $2x^2 + 5x - 3 = (2x - 1)(x + 3)$. The zeros are 2, $\frac{1}{2}$, and -3 .

12 An open-top box



- The box has height x and a square base of side $12 - 2x$, so $V(x) = x(12 - 2x)^2$, with $0 < x < 6$.
- Solve $x(12 - 2x)^2 = 128$. Expanding gives $4x^3 - 48x^2 + 144x - 128 = 0$, that is $x^3 - 12x^2 + 36x - 32 = 0$. Testing $x = 2$: $8 - 48 + 72 - 32 = 0$. Synthetic division gives $x^2 - 10x + 16 = (x - 2)(x - 8)$. The zeros are 2 and 8. Since x must lie in $(0, 6)$, only $x = 2$ works. Check: base 8×8 , height 2, volume 128. So the corners are squares of side 2 inches.

13 Hole or asymptote?



$f(x) = \frac{2(x-2)(x+2)}{(x-3)(x+2)}$. The factor $x+2$ cancels, so there is a **hole** at $x = -2$, at height $\frac{2(-4)}{-5} = \frac{8}{5}$.

The vertical asymptote is $x = 3$. The degrees are equal and $\frac{2}{1} = 2$, so the horizontal asymptote is $y = 2$. The x -intercept is 2 (the value -2 is the hole). The y -intercept is $f(0) = \frac{-8}{-6} = \frac{4}{3}$.

14 Sign chart for a quotient



Critical numbers: -4 (included, numerator 0) and 2 (excluded, denominator 0). Signs on $(-\infty, -4)$, $(-4, 2)$, $(2, \infty)$: $\frac{-}{-} = +$, $\frac{+}{-} = -$, $\frac{+}{+} = +$. The solution is $(-\infty, -4] \cup (2, \infty)$.

15 Slant asymptote



The numerator has degree 2 and the denominator degree 1, so there is a slant asymptote. Divide with $c = -2$:

$c = -2$	2	3	-5
Multiply by c		-4	2
Add	2	-1	-3

So $f(x) = 2x - 1 - \frac{3}{x+2}$. The slant asymptote is $y = 2x - 1$, and the vertical asymptote is $x = -2$.

16 Build a polynomial



Since the coefficients are real, $2 + i$ is also a zero, and

$(x - (2 - i))(x - (2 + i)) = x^2 - 4x + 5$. So $P(x) = a(x + 3)(x - 1)^2(x^2 - 4x + 5)$.

$P(0) = a \cdot 3 \cdot 1 \cdot 5 = 15a = 30$, so $a = 2$. Thus $P(x) = 2(x + 3)(x - 1)^2(x^2 - 4x + 5)$.

The degree is $1 + 2 + 2 = 5$ and the leading coefficient is $2 > 0$: the graph falls on the left and rises on the right.

17 From a graph to an equation



a. The graph touches the axis at -1 (multiplicity 2) and crosses at 2 (multiplicity 1):

$$f(x) = a(x + 1)^2(x - 2).$$

b. $f(0) = a \cdot 1 \cdot (-2) = 2$, so $a = -1$.

c. $(x + 1)^2(x - 2) = (x^2 + 2x + 1)(x - 2) = x^3 - 3x - 2$, so

$f(x) = -x^3 + 3x + 2$. Check: $f(2) = -8 + 6 + 2 = 0$. The negative leading coefficient also matches a graph that falls on the right.

18 Zero given, find the rest



The coefficients are real, so $-i$ is also a zero, and $(x - i)(x + i) = x^2 + 1$ is a factor.

Dividing, $P(x) = (x^2 + 1)(x^2 + x - 6)$, since

$$(x^2 + 1)(x^2 + x - 6) = x^4 + x^3 - 5x^2 + x - 6. \text{ Then } x^2 + x - 6 = (x + 3)(x - 2).$$

The zeros are i , $-i$, 2 , and -3 .

19 A rational inequality with a twist

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The sign of $x - 2$ is unknown, so multiplying could reverse the inequality. Instead subtract:

$$\frac{x+1}{x-2} - 2 = \frac{x+1-2(x-2)}{x-2} = \frac{5-x}{x-2} \geq 0.$$

Critical numbers: **5** (included) and **2** (excluded). Signs on $(-\infty, 2)$, $(2, 5)$, $(5, \infty)$: $\frac{+}{-} = -$, $\frac{+}{+} = +$, $\frac{-}{+} = -$. The solution is $(2, 5]$. Check: at $x = 5$ the fraction is $6/3 = 2$, and at $x = 3$ it is $4 \geq 2$.

20 A quartic inequality

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Factor as a quadratic in x^2 :

$x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4) = (x - 1)(x + 1)(x - 2)(x + 2)$. Critical numbers: $-2, -1, 1, 2$. Signs on the five intervals: $+, -, +, -, +$ (test $x = -3, -1.5, 0, 1.5, 3$). We want the product ≤ 0 , and the zeros are included. The solution is $[-2, -1] \cup [1, 2]$.

21 Average cost

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a. The degrees are equal, so the horizontal asymptote is $y = \frac{8}{1} = 8$. As production grows, the average cost gets closer and closer to 8 dollars per item, the materials cost, as the fixed cost is spread over more items.

b. Solve $\frac{8x + 1200}{x} \leq 10$. Since $x > 0$, multiplying by x is safe: $8x + 1200 \leq 10x$, so $x \geq 600$. Check: $A(600) = \frac{6000}{600} = 10$. At least 600 items are needed.

22

Missing term, fractional divisor

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The x^2 term is missing, so use coefficients 2, 0, -5 , 1.

$$c = \frac{1}{2}$$

2

0

 -5

1

Multiply by c

1

 $\frac{1}{2}$ $-\frac{9}{4}$

Add

2

1

 $-\frac{9}{2}$ $-\frac{5}{4}$

The quotient is $2x^2 + x - \frac{9}{2}$ and the remainder is $-\frac{5}{4}$, so $P(\frac{1}{2}) = -\frac{5}{4}$. Check directly:
 $2 \cdot \frac{1}{8} - \frac{5}{2} + 1 = \frac{1}{4} - \frac{5}{2} + 1 = -\frac{5}{4}$. Because the remainder is not 0, $x - \frac{1}{2}$ is not a factor.



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