

Functions and Transformations: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Evaluating a function



Substitute each input in parentheses.

$$f(-2) = 3(4) - 2(-2) + 5 = 12 + 4 + 5 = 21.$$

$$f(0) = 0 - 0 + 5 = 5.$$

$$f(3) = 3(9) - 6 + 5 = 27 - 6 + 5 = 26.$$

Answer: $f(-2) = 21$, $f(0) = 5$, $f(3) = 26$.

2 Is it a function?



a) In **A**, every first coordinate appears once, so each input has one output: **A** is a function. (Two inputs may share an output, as **-1** and **0** do.) In **B**, the input 1 is paired with both 3 and **-3**, so **B** is **not** a function.

b) Domain of **A**: $\{-3, -1, 0, 2, 4\}$. Range: $\{-5, 1, 2, 4\}$ (the value 2 is listed once).

3 Finding domains



- a) Need $x - 4 \geq 0$, so $x \geq 4$.
- b) The denominator cannot be zero: $x + 3 \neq 0$, so $x \neq -3$.
- c) Need $3x + 12 \geq 0$, so $3x \geq -12$ and $x \geq -4$.

4 Name the shift



- a) Parent x^2 : shifted 4 units left and 7 units down. Vertex $(-4, -7)$.
- b) Parent $|x|$: reflected over the x-axis, then shifted 2 units up. Vertex $(0, 2)$, opening downward.
- c) Parent $\sqrt{x}$: shifted 5 units right. The starting point is $(5, 0)$.

5 Sum, difference, product



$$(f + g)(x) = 2x + 3 + x - 6 = 3x - 3.$$
$$(f - g)(x) = 2x + 3 - x + 6 = x + 9.$$
$$(fg)(x) = (2x + 3)(x - 6) = 2x^2 - 12x + 3x - 18 = 2x^2 - 9x - 18.$$
$$(f + g)(4) = 3(4) - 3 = 9. \text{ Check: } f(4) + g(4) = 11 + (-2) = 9.$$

6 First composition



- a) $g(3) = 6$, so $f(g(3)) = f(6) = 11$. $f(3) = 8$, so $g(f(3)) = g(8) = 16$.
- b) $f(g(x)) = 2x + 5$ and $g(f(x)) = 2(x + 5) = 2x + 10$. They are not equal (they differ by 5), so the order matters.

7 A first inverse



Write $y = 4x - 7$ and swap: $x = 4y - 7$. Solve: $4y = x + 7$, so $y = \frac{x+7}{4}$. Thus

$$f^{-1}(x) = \frac{x+7}{4}.$$

Check: $f^{-1}(5) = \frac{12}{4} = 3$, and indeed $f(3) = 4(3) - 7 = 5$.

8 Piecewise evaluation



Find the piece for each input.

$$-3 < -1: f(-3) = -3 + 4 = 1.$$

$$x = -1 \text{ belongs to the middle piece: } f(-1) = (-1)^2 = 1.$$

$$f(0) = 0^2 = 0.$$

$$x = 2 \text{ belongs to the middle piece (since } 2 \leq 2): f(2) = 4.$$

$$5 > 2: f(5) = 6 - 5 = 1.$$

9 Build the equation



a) Reflection: $-x^2$. Shift right 3: replace x by $x - 3$. Shift up 5: add 5. So

$$g(x) = -(x - 3)^2 + 5.$$

b) The vertex is $(3, 5)$. $g(1) = -(1 - 3)^2 + 5 = -4 + 5 = 1$.

10 Taxi fare



- a) A distance cannot be negative: domain $m \geq 0$. The smallest fare is $C(0) = 3$, so the range is $C \geq 3$.
- b) $C(12) = 3 + 2.5(12) = 33$: the fare is \$33.
- c) Solve $3 + 2.5m = 48$: $2.5m = 45$, so $m = 18$ miles.
- d) Swap and solve: $m = \frac{C - 3}{2.5}$, so $C^{-1}(x) = \frac{x - 3}{2.5}$. It converts a fare in dollars into the distance in miles that was driven (check: $C^{-1}(48) = 18$).

11 Composition with a quadratic



- $f(g(x)) = (2x + 1)^2 - 3 = 4x^2 + 4x + 1 - 3 = 4x^2 + 4x - 2$.
- $g(f(x)) = 2(x^2 - 3) + 1 = 2x^2 - 5$.
- $f(g(2)) = f(5) = 25 - 3 = 22$; the formula gives $16 + 8 - 2 = 22$.
- $g(f(2)) = g(1) = 3$; the formula gives $8 - 5 = 3$.

12 A quotient with a hole



- a) $\frac{x^2 - 9}{x - 3} = \frac{(x - 3)(x + 3)}{x - 3} = x + 3$. The domain still excludes the value that makes $g(x) = 0$: $x \neq 3$. The graph is the line $y = x + 3$ with a hole at $(3, 6)$.
- b) $\frac{g}{f} = \frac{1}{x + 3}$ after simplifying, but $f(x) = 0$ when $x = 3$ or $x = -3$. The domain is $x \neq 3$ and $x \neq -3$.

13 Inverse of a rational function



Write $y = \frac{3}{x-1} + 2$ and swap: $x = \frac{3}{y-1} + 2$. Then $x - 2 = \frac{3}{y-1}$, so $y - 1 = \frac{3}{x-2}$ and $y = \frac{3}{x-2} + 1$. Thus $f^{-1}(x) = \frac{3}{x-2} + 1$.

Check: $f(4) = \frac{3}{3} + 2 = 3$ and $f^{-1}(3) = \frac{3}{1} + 1 = 4$.

14 Absolute value equation and inequality



a) $2x - 5 = 9$ gives $x = 7$; $2x - 5 = -9$ gives $x = -2$. Check: $|2(-2) - 5| = |-9| = 9$.

b) $|x - 3| < 4$ means “the distance from x to 3 is less than 4”, so $-4 < x - 3 < 4$, that is $-1 < x < 7$. Interval notation: $(-1, 7)$.

15 Domain and range from a transformation



a) The radicand must be nonnegative: $x + 3 \geq 0$, so the domain is $x \geq -3$. Since $\sqrt{x+3} \geq 0$, we have $-2\sqrt{x+3} \leq 0$, hence $g(x) \leq 1$: the range is $y \leq 1$.

b) $g(1) = -2\sqrt{4} + 1 = -3$. For $g(x) = -5$: $-2\sqrt{x+3} = -6$, so $\sqrt{x+3} = 3$ and $x + 3 = 9$, giving $x = 6$. Check: $g(6) = -2(3) + 1 = -5$.

c) The parent $\sqrt{x}$ is shifted 3 units left, stretched vertically by 2, reflected over the x-axis and shifted 1 unit up. The curve starts at $(-3, 1)$ and falls, as in the figure.

16 Inverse with a restricted domain



- a) Without a restriction the parabola fails the horizontal line test (for instance $f(2) = f(4) = 2$), so it has no inverse function. For $x \geq 3$ the function is one-to-one.
- b) Swap: $x = (y - 3)^2 + 1$, so $(y - 3)^2 = x - 1$. Because $y \geq 3$, take the positive root: $y - 3 = \sqrt{x - 1}$. Thus $f^{-1}(x) = \sqrt{x - 1} + 3$. Its domain is the range of f , namely $x \geq 1$; its range is the domain of f , namely $y \geq 3$.
- c) $f(5) = 4 + 1 = 5$ and $f^{-1}(5) = \sqrt{4} + 3 = 5$. The point $(5, 5)$ lies on $y = x$, so it is its own mirror image.

17 Does the order matter?



- a) $f(g(x)) = \sqrt{x - 4}$ and $g(f(x)) = \sqrt{x} - 4$.
- b) For $f(g(x))$ we need $x - 4 \geq 0$: domain $x \geq 4$. For $g(f(x))$ we need $x \geq 0$: domain $x \geq 0$.
- c) They are different functions, with different domains. For example $f(g(5)) = \sqrt{1} = 1$ but $g(f(5)) = \sqrt{5} - 4 \approx -1.76$; and $x = 1$ is allowed in the second but not in the first.

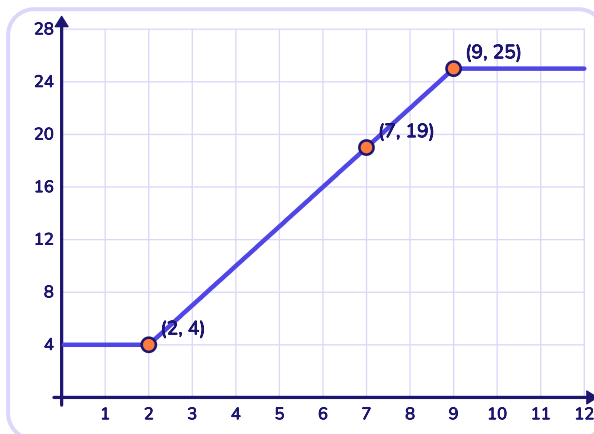
18 Parking fees

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a) $P(1.5) = 4$. $P(5) = 4 + 3(3) = 13$. $P(12) = 25$.

b) $4 + 3(t - 2) = 19$ gives $3(t - 2) = 15$, so $t = 7$ hours.

c) At $t = 9$: $4 + 3(7) = 25$, exactly the cap, so the pieces connect at $(9, 25)$. The graph is a flat segment, a rising segment, then a flat segment again.



19 Absolute value as a piecewise function

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a) If $x \geq 1$, then $|x - 1| = x - 1$ and $h(x) = 2x - 2 - 3 = 2x - 5$. If $x < 1$, then $|x - 1| = 1 - x$ and $h(x) = 2 - 2x - 3 = -2x - 1$.

b) $2|x - 1| = 3$ gives $|x - 1| = 1.5$, so $x = 2.5$ or $x = -0.5$. Check:
 $h(2.5) = 2(1.5) - 3 = 0$.

c) The vertex is $(1, -3)$, the graph opens upward, so the range is $y \geq -3$.

20 Discount or coupon first?

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a) Discount first: $f(60) = 48$, then $g(48) = 43$. Coupon first: $g(60) = 55$, then $f(55) = 44$.

b) $g(f(x)) = 0.8x - 5$ and $f(g(x)) = 0.8(x - 5) = 0.8x - 4$. Applying the discount first is always cheaper, by exactly \$1, whatever the price (as long as the price is large enough to use the coupon).

21 Transforming a table of values



a) The input shift $x + 1$ moves each point 1 unit left; the outputs are doubled, then lowered by 3. The point (a, b) becomes $(a - 1, 2b - 3)$: $(-2, 1) \rightarrow (-3, -1)$; $(0, 3) \rightarrow (-1, 3)$; $(1, -2) \rightarrow (0, -7)$; $(4, 0) \rightarrow (3, -3)$.

b) $f(2x)$ compresses the graph horizontally by the factor $\frac{1}{2}$: $(a, b) \rightarrow (a/2, b)$: $(-1, 1)$, $(0, 3)$, $(0.5, -2)$, $(2, 0)$.

22 Proving two functions are inverses



$f(g(x)) = \frac{2 \cdot \frac{3x+1}{x-2} + 1}{\frac{3x+1}{x-2} - 3}$. Multiply the numerator and denominator by $x - 2$:

numerator: $2(3x + 1) + (x - 2) = 7x$; denominator: $(3x + 1) - 3(x - 2) = 7$.

So $f(g(x)) = \frac{7x}{7} = x$ (for $x \neq 2$). This shows that f undoes g . To be fully sure that they are inverses, also verify $g(f(x)) = x$: $\frac{3(2x + 1) + (x - 3)}{(2x + 1) - 2(x - 3)} = \frac{7x}{7} = x$. Both hold, so $g = f^{-1}$.



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