

Functions and Transformations: math lesson, Grade 11 – download the PDF

Algebra 2 is built on one big idea: a **function** is a machine you can study, reshape, combine and run backwards. In this chapter you will review how functions are written and read, meet the “parent” graphs that every other graph grows from, and learn to shift, stretch, flip, add, compose and invert them. Think of it as your toolkit for the rest of the year.

1. Function notation

D Function

A **function** assigns to each input exactly one output. We write $y = f(x)$, read “ f of x ”. The symbol $f(x)$ is **not** “ f times x ”: it is the output produced when the input is x .

To evaluate a function, replace every x by the input, using parentheses, then simplify. An input can be a number, a letter or even an expression. A graph represents a function exactly when no vertical line crosses it more than once (the **vertical line test**).

E Example 1: evaluating

Let $f(x) = -x^2 + 3x + 2$.

$$f(-1) = -(-1)^2 + 3(-1) + 2 = -1 - 3 + 2 = -2.$$

$$f(a+1) = -(a+1)^2 + 3(a+1) + 2 = -a^2 - 2a - 1 + 3a + 3 + 2 = -a^2 + a + 4.$$

Quick check with $a = 1$: $f(2) = -4 + 6 + 2 = 4$ and $-1 + 1 + 4 = 4$. It matches.

2. Domain and range

D Domain and range

The **domain** is the set of all allowed inputs (x -values). The **range** is the set of all outputs (y -values) the function actually produces.

M Finding a domain from a formula

1. Start with all real numbers.
2. Remove any x that makes a **denominator zero**.
3. Remove any x that puts a **negative number under a square root** (or any even root).
4. Write the answer as an inequality or in interval notation.

E Example 2: domain and range

Let $f(x) = \sqrt{x-2} + 3$. The radicand must satisfy $x - 2 \geq 0$, so the domain is $x \geq 2$, or $[2, \infty)$. Since $\sqrt{x-2} \geq 0$, we get $f(x) \geq 3$: the range is $y \geq 3$, or $[3, \infty)$. The smallest output, 3, happens at $x = 2$.

⚠ Watch out

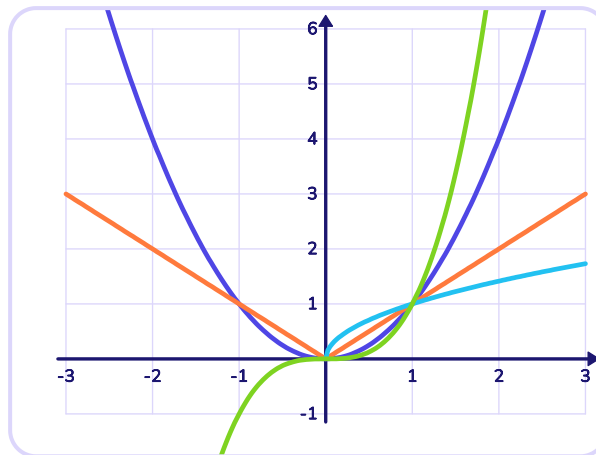
A real-life situation can shrink the domain further. If t is a number of hours, then $t \geq 0$ even when the formula would accept negative numbers.

3. Parent functions

A **parent function** is the simplest member of a family. Learn these six shapes by heart: almost every graph you meet this year is a transformed copy of one of them.

Parent function	Rule	Domain	Range
Linear	$f(x) = x$	all reals	all reals
Quadratic	$f(x) = x^2$	all reals	$y \geq 0$
Cubic	$f(x) = x^3$	all reals	all reals
Absolute value	$f(x) = x $	all reals	$y \geq 0$
Square root	$f(x) = \sqrt{x}$	$x \geq 0$	$y \geq 0$
Reciprocal	$f(x) = \frac{1}{x}$	$x \neq 0$	$y \neq 0$

Here are four of them on the same grid: the parabola x^2 (violet), the V-shaped $|x|$ (orange), the square root (cyan) and the cubic x^3 (green).



4. Transformations of graphs

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Transformation rules

Starting from the graph of $y = f(x)$, the graph of $g(x) = a f(x - h) + k$ is obtained by:

- a **horizontal shift** of h units (right if $h > 0$, left if $h < 0$);
- a **vertical stretch** by the factor $|a|$ if $|a| > 1$, or a **vertical compression** if $0 < |a| < 1$, plus a **reflection over the x-axis** if $a < 0$;
- a **vertical shift** of k units (up if $k > 0$, down if $k < 0$).

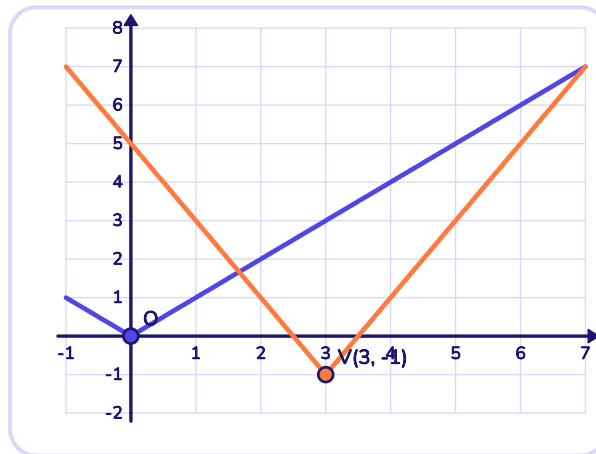
Two more moves: $f(-x)$ reflects over the y-axis, and $f(bx)$ compresses horizontally by the factor $\frac{1}{b}$ when $b > 1$.

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Example 3: reading a transformation

Take $g(x) = -2(x - 1)^2 + 8$. The parent is x^2 . The graph is shifted 1 unit right and 8 units up, stretched vertically by 2 and reflected over the x-axis, so it opens downward. The vertex is $(1, 8)$. To find the x-intercepts, solve $-2(x - 1)^2 + 8 = 0$: then $(x - 1)^2 = 4$, so $x = 3$ or $x = -1$.

The picture below shows the parent $y = |x|$ and its image $g(x) = 2|x - 3| - 1$: the vertex moves from $(0, 0)$ to $(3, -1)$ and the V becomes twice as steep.



Watch out

The sign inside the parentheses works backwards: $f(x + 3)$ moves the graph **left** 3 units, because you need $x = -3$ to get the same input as $f(0)$.

5. Combining functions

D Operations on functions

For two functions f and g : $(f + g)(x) = f(x) + g(x)$, $(f - g)(x) = f(x) - g(x)$,
 $(fg)(x) = f(x) \cdot g(x)$ and $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with $g(x) \neq 0$.

The domain of a sum, difference or product is the set of x -values that are allowed in **both** functions. For a quotient, also remove every x with $g(x) = 0$.

E Example 4: sum and quotient

Let $f(x) = \sqrt{x}$ and $g(x) = x - 4$. Then $(f + g)(9) = 3 + 5 = 8$. The domain of $f + g$ is $x \geq 0$. For $\frac{f}{g}$, we need $x \geq 0$ and $x \neq 4$, so the domain is $[0, 4) \cup (4, \infty)$.

6. Composition of functions

D Composition

The **composition** of f and g is $(f \circ g)(x) = f(g(x))$: feed x into g first, then feed the result into f . Its domain contains the x in the domain of g for which $g(x)$ lies in the domain of f .

M**Composing step by step**

1. Write the **inner** function (the one closest to x).
2. Substitute that whole expression, in parentheses, for every x of the outer function.
3. Simplify, then check with a test value.

E**Example 5: both orders**

Let $f(x) = x^2 + 1$ and $g(x) = 3x - 2$.

$$f(g(x)) = (3x - 2)^2 + 1 = 9x^2 - 12x + 5 \text{ and}$$

$$g(f(x)) = 3(x^2 + 1) - 2 = 3x^2 + 1.$$

Test with $x = 1$: $f(g(1)) = f(1) = 2$ and $9 - 12 + 5 = 2$; $g(f(1)) = g(2) = 4$ and $3 + 1 = 4$.

**Watch out**

Composition is **not commutative**: in general $f(g(x)) \neq g(f(x))$, as Example 5 shows.

7. Inverse functions

D Inverse function

If f is **one-to-one** (each output comes from one input only, so no horizontal line crosses its graph twice), its **inverse** f^{-1} undoes f : $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

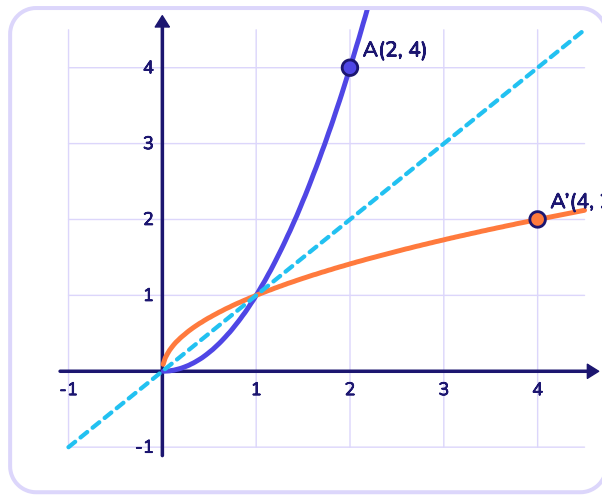
M Finding an inverse

1. Write $y = f(x)$.
2. Swap x and y .
3. Solve for y .
4. Name the result $f^{-1}(x)$ and check with a composition.

E Example 6: a rational function

Let $f(x) = \frac{2x}{x-1}$. Swap: $x = \frac{2y}{y-1}$, so $x(y-1) = 2y$, then $xy - x = 2y$, then $y(x-2) = x$. Hence $f^{-1}(x) = \frac{x}{x-2}$. The domain and range swap: f has domain $x \neq 1$ and range $y \neq 2$, while f^{-1} has domain $x \neq 2$ and range $y \neq 1$. Check: $f(2) = 4$ and $f^{-1}(4) = \frac{4}{4-2} = 2$.

Graphically, the graphs of f and f^{-1} are mirror images over the line $y = x$: the point (a, b) on f becomes (b, a) on f^{-1} . The parabola $y = x^2$ fails the horizontal line test, so we restrict it to $x \geq 0$; its inverse is then $\sqrt{x}$.



Watch out

The notation f^{-1} does **not** mean $\frac{1}{f}$. For $f(x) = 5x - 2$, the inverse is $\frac{x+2}{5}$, not $\frac{1}{5x-2}$.

8. Piecewise and absolute value functions



Piecewise function

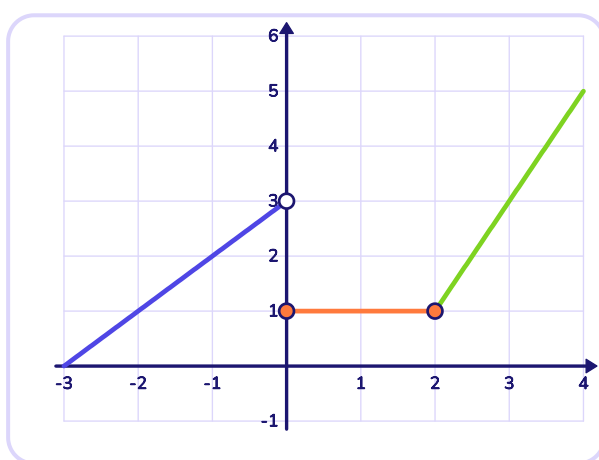
A **piecewise function** uses different rules on different parts of its domain. To evaluate it, first decide which interval the input belongs to, then use that rule only.

The absolute value is the most famous piecewise function: $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$. Geometrically, $|x - c|$ is the distance between x and c on the number line.

E Example 7: solving an absolute value equation

Solve $|2x + 1| = 7$. The quantity inside is 7 or -7 . So $2x + 1 = 7$ gives $x = 3$, and $2x + 1 = -7$ gives $x = -4$. Check: $|2(3) + 1| = 7$ and $|2(-4) + 1| = |-7| = 7$.

Here is a piecewise function with three rules: $x + 3$ for $x < 0$, the constant 1 for $0 \leq x \leq 2$, and $2x - 3$ for $x > 2$. The open circle at $(0, 3)$ means that point is **not** on the graph; the filled circle at $(0, 1)$ is.



! Zyro's tip

On my home planet we say: "Before you plug in, find the piece." Circle the interval that contains your input, and only then pick the rule.

Key takeaways

- A function gives exactly one output per input; $f(x)$ means “the output for the input x ”.
- Domain: remove zero denominators and negative radicands. Range: read it from the graph or from the transformation.
- For $g(x) = a f(x - h) + k$: shift h right, k up, stretch by $|a|$, reflect over the x-axis if $a < 0$.
- $(f \circ g)(x) = f(g(x))$: inner function first. Order matters.
- To invert: swap x and y , then solve. Domain and range swap; the graph reflects over $y = x$.
- Piecewise functions: pick the right piece first. $|x - c|$ is a distance and splits into two cases.



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