

Relations and Functions: math lesson, Grade 9 – download the PDF

A vending machine, a taxi meter, a phone plan: in each one, a number you choose decides another number you get back. Mathematicians call these links **relations**, and the best-behaved ones **functions**. In this chapter you will learn to describe them with pairs, tables, mapping diagrams, graphs, and the famous notation $f(x)$.

1. Relations, domain, and range

D Relation

A **relation** is a set of **ordered pairs** (x, y) . The set of all first coordinates is the **domain** (the inputs). The set of all second coordinates is the **range** (the outputs).

A relation can be written as a list of pairs, shown in a table, drawn as a mapping diagram, or plotted as points on the coordinate plane. When you list the domain and range, write each value once and in increasing order.

E Example 1: domain and range

Let $R = \{(1, 4), (2, 6), (3, 6), (5, 9)\}$.

The first coordinates are **1, 2, 3, 5**, so the domain is $\{1, 2, 3, 5\}$.

The second coordinates are **4, 6, 6, 9**. The value **6** appears twice but is listed once, so the range is $\{4, 6, 9\}$.

I 2. What is a function?

D Function

A **function** is a relation in which **every input has exactly one output**. In other words, no x -value is ever paired with two different y -values.

Notice what the definition allows: two different inputs may share the same output. What it forbids is one input giving two different outputs, just like a vending machine that must give you exactly one snack for each button.

E Example 2: function or not?

$A = \{(2, 5), (3, 5), (4, 8)\}$ is a function: the inputs **2, 3, 4** each appear once. The repeated output **5** is fine.

$B = \{(2, 5), (2, 7), (4, 8)\}$ is **not** a function: the input **2** is paired with both **5** and **7**.



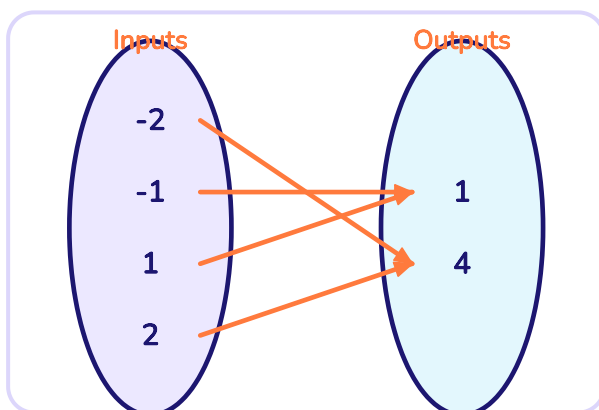
Common mistake

Students often reject a relation because an *output* repeats. That is allowed! Only a repeated *input* with different outputs breaks the rule.

3. Tables, mappings, and graphs

The same relation can wear several outfits. Here is the function that squares its input, for the inputs $-2, -1, 1, 2$, as a table and as a mapping diagram.

x	-2	-1	1	2
y	4	1	1	4



In a mapping diagram, check the arrows leaving the input oval: if each input has **one** arrow, it is a function. Two arrows can land on the same output, but one input can never send out two arrows.

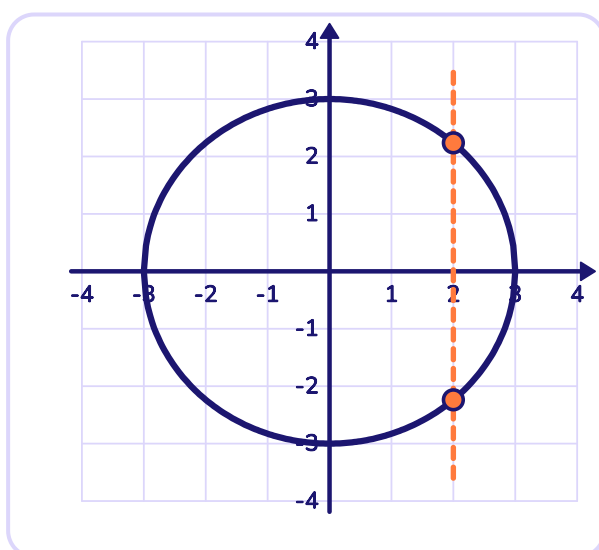
M**Method: is this relation a function?**

1. List the inputs (the domain).
2. Look for an input that appears more than once.
3. If it does, compare its outputs: different outputs means **not a function**.
4. If no input has two different outputs, it is a function.

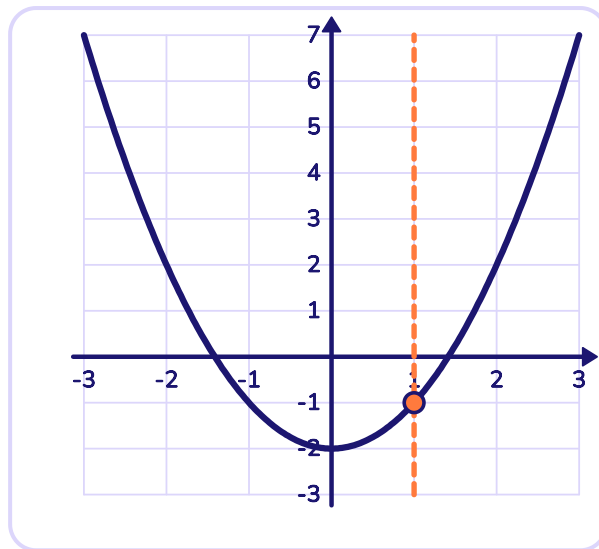
4. The vertical line test**P****Vertical line test**

A graph represents a function if and only if **no vertical line crosses it more than once**.

Why does it work? A vertical line sits at one fixed x -value. If it touches the graph twice, that input has two outputs.



The circle fails: the line $x = 2$ touches it at two points. The parabola below passes: every vertical line meets it at most once.



5. Function notation

Instead of writing $y = 3x - 5$, we give the function a name, often f , and write $f(x) = 3x - 5$. Read it as “**f of x**”.

D Function notation

$f(x)$ is the **output** of the function f when the input is x . So $f(4)$ means “the output when the input is 4”, and the point $(4, f(4))$ lies on the graph.

⚠ Careful

$f(x)$ does **not** mean f times x . The parentheses hold the input. Other letters work too: $g(t)$, $h(n)$, $C(m)$.



Zyro's tip

On my planet we call a function a “machine”: drop an input in the top, and exactly one output falls out the bottom. Picture it whenever $f(x)$ looks scary!

6. Evaluating functions

To **evaluate** a function, replace the variable by the given input (use parentheses!) and simplify.



Example 3: evaluating

Let $g(x) = x^2 - 4x + 1$.

$$g(3) = 3^2 - 4(3) + 1 = 9 - 12 + 1 = -2.$$

$$g(-1) = (-1)^2 - 4(-1) + 1 = 1 + 4 + 1 = 6.$$



Example 4: finding the input

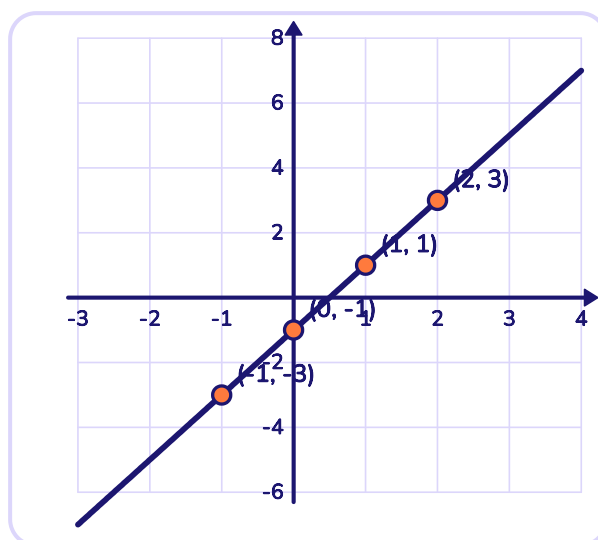
Let $f(x) = 3x - 5$. Find x so that $f(x) = 19$.

Solve $3x - 5 = 19$, so $3x = 24$ and $x = 8$. Check: $f(8) = 24 - 5 = 19$.

7. Graphs of functions

The **graph** of f is the set of all points $(x, f(x))$. To draw it, make a table of inputs and outputs, plot the points, and connect them when the domain is all real numbers.

x	-1	0	1	2
$f(x) = 2x - 1$	-3	-1	1	3



Reading a graph goes the other way: to find $f(2)$, start at $x = 2$ on the horizontal axis, go up to the curve, then across to the vertical axis. The **domain** is the set of x -values the graph covers, and the **range** is the set of y -values it reaches.

8. Arithmetic sequences as functions

A **sequence** is a list of numbers; it is a function whose inputs are the positions $n = 1, 2, 3, \dots$. In an **arithmetic sequence**, you add the same number d , the **common difference**, to get from one term to the next.

P Explicit rule

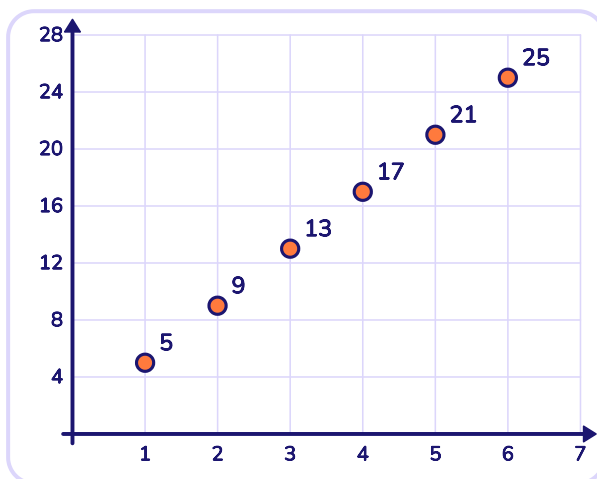
If the first term is a_1 and the common difference is d , then

$$a_n = a_1 + (n - 1)d.$$

E Example 5: an arithmetic sequence

For **5, 9, 13, 17, ...** we have $a_1 = 5$ and $d = 4$, so $a_n = 5 + 4(n - 1) = 4n + 1$.

The tenth term is $a_{10} = 41$.



The graph is made of separate points, not a connected line, because n must be a positive whole number. The points lie on a straight line of slope $d = 4$.

E Example 6: using two terms

An arithmetic sequence has $a_3 = 14$ and $a_7 = 30$. Four steps separate them, so

$d = \frac{30 - 14}{4} = 4$. Then $a_1 = 14 - 2 \cdot 4 = 6$ and $a_n = 6 + 4(n - 1) = 4n + 2$.

Check: $a_7 = 28 + 2 = 30$.

Key takeaways

- A relation is a set of ordered pairs; the domain holds the inputs and the range holds the outputs.
- A function gives each input exactly one output. Repeated outputs are fine.
- Vertical line test: a graph is a function if no vertical line crosses it twice.
- $f(x)$ is the output for input x . To evaluate, substitute with parentheses.
- To solve $f(x) = k$, set the rule equal to k and solve for x .
- An arithmetic sequence is a function of n : $a_n = a_1 + (n - 1)d$.



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