

Introduction to Functions: math lesson, Grade 8 – download the PDF

Every time you look up a price, check the temperature, or watch a game score change, one quantity depends on another. Functions give us a precise way to describe that dependence. In this chapter you will learn what a function is, how to show it in four different ways, how to tell lines from curves, and how to compare two functions or sketch one from a story.

1. What is a function?

Think of a machine: you drop in a number, the machine follows a rule, and a number comes out. The number going in is the **input**, the number coming out is the **output**.

D Definition: function

A **function** is a rule that assigns to each input exactly one output. We often call the input x and the output y , and we say that y is a function of x .

The key words are *exactly one*. An input may never lead to two different outputs. However, two different inputs are allowed to give the same output.

E Example 1: a machine rule

A machine uses the rule $y = 3x + 1$. The input $x = 0$ gives $y = 3(0) + 1 = 1$, the input $x = 1$ gives $y = 4$, the input $x = 2$ gives $y = 7$ and the input $x = 5$ gives $y = 16$. Each input has exactly one output, so the rule is a function.

Input x	0	1	2	5
Output y	1	4	7	16

⚠ Common mistake

Many students think that two inputs sharing an output breaks the rule. It does not. The pairs $(2, 5)$ and $(3, 5)$ are fine. What is forbidden is $(2, 5)$ together with $(2, 8)$, because the input 2 would have two outputs.

I 2. Four ways to represent a function

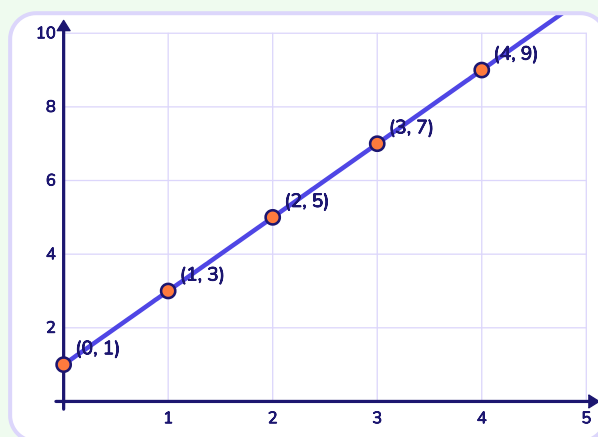
The same function can be described in four ways, and a good mathematician moves easily between them.

- **In words:** “A plant is 4 cm tall today and grows 2 cm each day.”
- **With a table:** a list of inputs next to their outputs.
- **With an equation:** $y = 2x + 4$, where x is the number of days.
- **With a graph:** the ordered pairs (x, y) plotted on the coordinate plane.

Day x	0	1	2	3
Height y (cm)	4	6	8	10

E Example 2: from an equation to a graph

Take $y = 2x + 1$. Choose inputs and compute outputs: $x = 0 \Rightarrow y = 1$, $x = 1 \Rightarrow y = 3$, $x = 2 \Rightarrow y = 5$, $x = 3 \Rightarrow y = 7$, $x = 4 \Rightarrow y = 9$. Plot the points $(0, 1)$, $(1, 3)$, $(2, 5)$, $(3, 7)$, $(4, 9)$. They line up, and the straight line through them is the graph.



M Method: graphing a function from its equation

1. Pick at least three inputs, including 0.
2. Compute each output with the rule.
3. Write the ordered pairs in a table.
4. Plot the points and connect them if the situation allows any value in between.

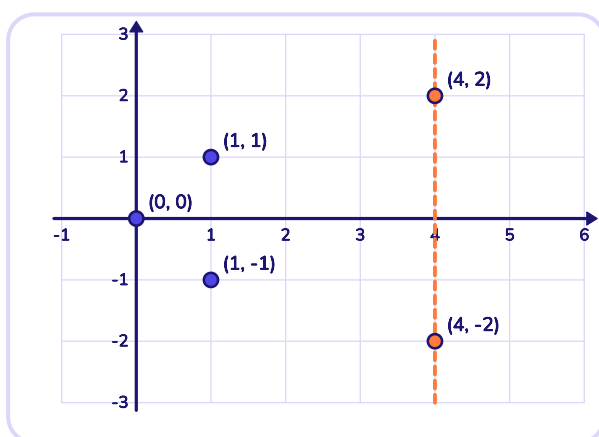
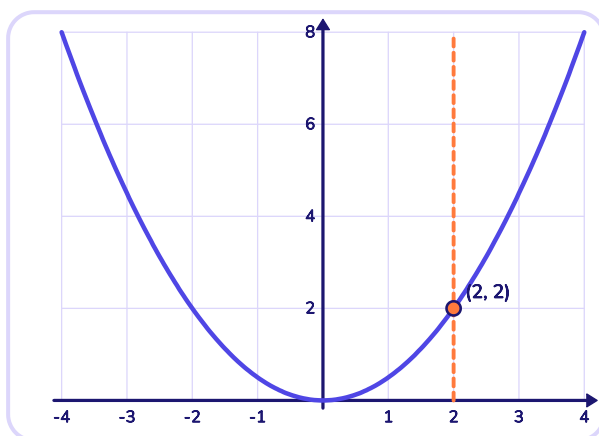
3. The vertical line test

Looking at a graph, how can you tell whether it shows a function? A graph shows a function when no input has two outputs. On a graph, one input is a vertical position. So a vertical line must never touch the graph twice.



Vertical line test

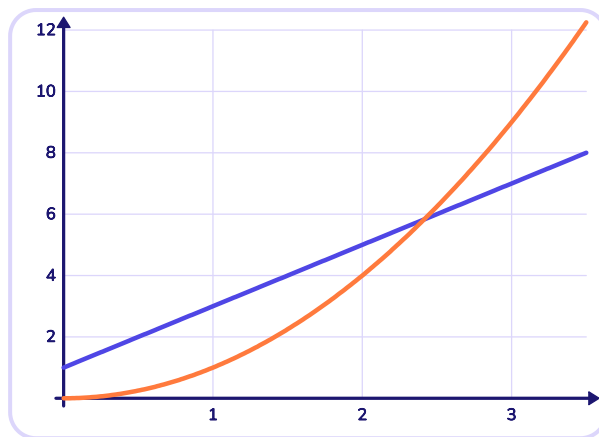
A graph represents a function if and only if every vertical line crosses the graph at most once.



In the first graph the dashed line $x = 2$ meets the curve once. In the second graph the dashed line $x = 4$ meets the points $(4, 2)$ and $(4, -2)$: the input 4 has two outputs, so this relation is *not* a function.

4. Linear versus nonlinear functions

A **linear function** has a graph that is a straight line. Its equation can be written $y = mx + b$. Every other function is **nonlinear**: its graph is a curve, or it is a line broken into pieces. For example $y = x^2$ and $y = x^3$ are nonlinear.



In a table with equally spaced inputs, a linear function has a *constant change* in the outputs. In a nonlinear function the changes vary.

E Example 3: testing a table

For $y = x^2$, the outputs go **0, 1, 4, 9, 16**. The changes are **+1, +3, +5, +7**. They are not constant, so $y = x^2$ is nonlinear.

x	0	1	2	3	4
$y = x^2$	0	1	4	9	16
Change in y		+1	+3	+5	+7

5. Rate of change and initial value

For a linear function $y = mx + b$, two numbers tell the whole story.

D Definition: rate of change and initial value

The **rate of change** m tells how much y changes when x increases by 1. It is found with $m = \frac{\text{change in } y}{\text{change in } x}$. The **initial value** b is the output when $x = 0$; it is where the graph crosses the y -axis.

E Example 4: finding the equation from two points

A plant is 4 cm tall on day 0 and 10 cm tall on day 3. The rate of change is $\frac{10 - 4}{3 - 0} = 2$ cm per day. The initial value is 4 cm. The equation is $y = 2x + 4$. On day 10 the height is $2(10) + 4 = 24$ cm, which is about 9.4 inches.

A positive rate means the function increases, a negative rate means it decreases, and a rate of 0 means it stays constant. In a real situation, always give the units: dollars per month, gallons per minute, miles per hour.

! Zyro's tip

On my planet we say: "the rate is the staircase, the initial value is the ground floor."
Start at the initial value, then climb one step of size m for every unit of x !

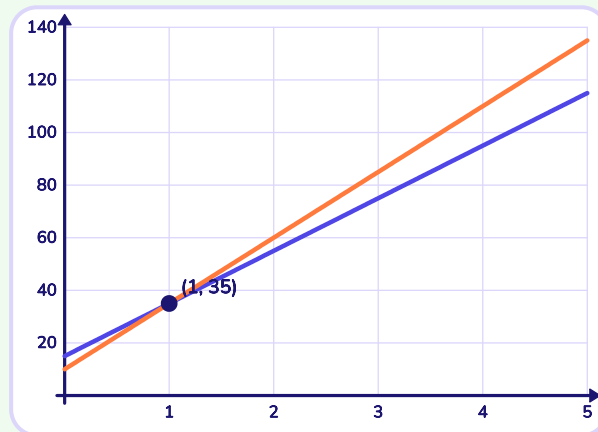
6. Comparing two functions

Two functions can be given in different forms: one by an equation, another by a table or a graph. To compare them, first find the rate of change and the initial value of each.

E**Example 5: two gym plans**

Plan A costs $y = 20x + 15$ dollars for x months. Plan B is shown by a table: 35 dollars after 1 month, 60 after 2 months, 85 after 3 months. The table has a constant change of 25, so the rate of plan B is 25 dollars per month. Going back one month, the initial value is $35 - 25 = 10$ dollars, so B is $y = 25x + 10$.

Plan A has the greater initial value (15 versus 10), but plan B has the greater rate (25 versus 20). The plans cost the same when $20x + 15 = 25x + 10$, that is $5 = 5x$, so $x = 1$. After that, plan B costs more.

**7. Sketching a graph from a description**

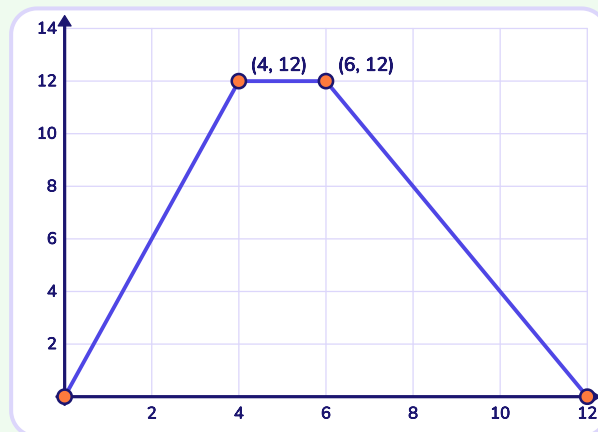
You do not always need numbers to draw a graph. Read the story one piece at a time and decide for each piece whether the graph goes up, goes down, or stays flat, and how steeply.

M**Method: sketching from a story**

1. Label the horizontal axis (usually time) and the vertical axis (the quantity), with units.
2. Mark the starting point.
3. For each stage, draw a segment: rising if the quantity grows, flat if it stays the same, falling if it shrinks.
4. Steeper segments mean faster changes.

E**Example 6: a bathtub**

A tub is empty. It fills at 3 inches per minute for 4 minutes, so the water reaches 12 inches. Nothing changes for 2 minutes. Then the tub drains at 2 inches per minute for 6 minutes until it is empty again. The points are $(0, 0)$, $(4, 12)$, $(6, 12)$, $(12, 0)$.



Key takeaways

- A function assigns to each input exactly one output.
- A function can be shown with words, a table, an equation or a graph.
- Vertical line test: a graph is a function if no vertical line crosses it more than once.
- A linear function has a straight-line graph, $y = mx + b$, with constant change in a table.
- The rate of change is $\frac{\text{change in } y}{\text{change in } x}$; the initial value is the output when $x = 0$.
- To compare functions, find the rate and the initial value of each, whatever the form.
- To sketch from a story, draw one segment per stage: up, flat or down.



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