

Function Composition and Inverses: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Evaluating composites



$$g(3) = 9 - 3 = 6, \text{ so } (f \circ g)(3) = f(6) = 17.$$

$$f(3) = 11, \text{ so } (g \circ f)(3) = g(11) = 121 - 3 = 118.$$

$$f(-1) = 3, \text{ so } (f \circ f)(-1) = f(3) = 11.$$

Answers: 17, 118 and 11.

2 Building composite formulas



$$(f \circ g)(x) = f(4x) = 4x - 7.$$

$$(g \circ f)(x) = g(x - 7) = 4(x - 7) = 4x - 28.$$

They are not equal (for $x = 1$ they give -3 and -24). Composition is not commutative.

3 A simple domain



$$(f \circ g)(x) = \sqrt{x + 6}. \text{ We need } x + 6 \geq 0, \text{ so the domain is } [-6, \infty).$$

$$(g \circ f)(x) = \sqrt{x} + 6. \text{ The inner function } \sqrt{x} \text{ requires } x \geq 0, \text{ so the domain is } [0, \infty).$$

4 Horizontal line test



f : a non-constant line, every horizontal line meets it once: one-to-one.

g : the line $y = 9$ meets the graph at $x = \pm 3$: not one-to-one.

h : increasing everywhere: one-to-one.

k : the line $y = 2$ meets it at $x = \pm 2$: not one-to-one.

m : each horizontal line $y = c$ with $c \neq 0$ meets it once at $x = 1/c$: one-to-one.

5 A linear inverse



Set $y = 5x - 8$. Then $5x = y + 8$ and $x = \frac{y+8}{5}$. So $f^{-1}(x) = \frac{x+8}{5}$.

Check: $f^{-1}(7) = \frac{15}{5} = 3$ and $f(3) = 15 - 8 = 7$.

6 Even, odd or neither



$f(-x) = x^6 - 2x^2 = f(x)$: even.

$g(-x) = -4x^3 - x = -g(x)$: odd.

$h(1) = 4$ and $h(-1) = 1 - 3 = -2$. Since -2 is neither 4 nor -4 , h is neither.

7 Absolute values and pieces



(a) $x + 4 = 9$ gives $x = 5$; $x + 4 = -9$ gives $x = -13$. Solutions: 5 and -13 .

(b) $q(-2) = 3(-2) - 1 = -7$. $q(2) = 3(2) - 1 = 5$ (the first rule, since $2 \leq 2$).

$q(4) = 16 - 1 = 15$.

8 Payroll at a repair shop



$$P(h(d)) = 22(6d) + 15 = 132d + 15.$$

For $d = 5$: $132(5) + 15 = 675$. Check: $h(5) = 30$ hours and $P(30) = 660 + 15 = 675$.

The technician earns \$675.

9 A hidden restriction



$$(f \circ g)(x) = f(x + 5) = \frac{1}{(x + 5) - 2} = \frac{1}{x + 3}.$$

The inner function g accepts every real number. The outer function needs $g(x) \neq 2$, that is $x + 5 \neq 2$, so $x \neq -3$.

Domain: all real numbers except -3 .

10 Inverse of a rational function



Set $y = \frac{4x - 1}{x + 3}$. Then $y(x + 3) = 4x - 1$, so $xy + 3y = 4x - 1$, $xy - 4x = -1 - 3y$ and $x(y - 4) = -(3y + 1)$. Hence $x = \frac{3y + 1}{4 - y}$.

$$f^{-1}(x) = \frac{3x + 1}{4 - x}, \text{ with } x \neq 4 \text{ (the value 4 is not in the range of } f).$$

$$\text{Check: } f(0) = -\frac{1}{3} \text{ and } f^{-1}\left(-\frac{1}{3}\right) = \frac{0}{4 + 1/3} = 0.$$

11 A cubic inverse



The cube function is increasing, so f is increasing and therefore one-to-one.

$$\text{Set } y = 2x^3 - 16. \text{ Then } x^3 = \frac{y + 16}{2} \text{ and } x = \sqrt[3]{\frac{y + 16}{2}}.$$

$$f^{-1}(x) = \sqrt[3]{\frac{x + 16}{2}}, \text{ defined for all real numbers. Check: } f(2) = 0 \text{ and}$$

$$f^{-1}(0) = \sqrt[3]{8} = 2.$$

12 Are they inverses?



$$f(g(x)) = 2 \cdot \frac{x-3}{2} + 3 = (x-3) + 3 = x.$$

$$g(f(x)) = \frac{(2x+3)-3}{2} = \frac{2x}{2} = x.$$

Both compositions give x , so f and g are inverses.

13 Reading a graph and its inverse



(a) Since $f(2) = 4$, $f^{-1}(4) = 2$. Since $f(-2) = -4$, $f^{-1}(-4) = -2$.

(b) Swap the coordinates: $(4, 2)$ and $(-4, -2)$.

(c) $y = \frac{x^3}{2}$ gives $x = \sqrt[3]{2y}$, so $f^{-1}(x) = \sqrt[3]{2x}$. Check: $f^{-1}(4) = \sqrt[3]{8} = 2$.

14 Describing a transformation



Shift right 3, reflect in the x-axis, shift up 5.

Vertex: $(3, 5)$, and the graph opens downward.

y-intercept: $g(0) = -|-3| + 5 = 2$.

x-intercepts: $|x-3| = 5$, so $x = 8$ or $x = -2$.

15 Symmetry proofs



$$f(-x) = \frac{-x}{(-x)^2 + 1} = -\frac{x}{x^2 + 1} = -f(x): \text{odd.}$$

$$g(-x) = |-x| + (-x)^2 = |x| + x^2 = g(x): \text{even.}$$

$h(1) = 2$ and $h(-1) = -1 + 1 = 0$. Since $0 \neq 2$ and $0 \neq -2$, h is neither even nor odd.

16 Absolute value as pieces



(a) $3 - 2x \geq 0$ when $x \leq \frac{3}{2}$. So $|3 - 2x| = 3 - 2x$ if $x \leq \frac{3}{2}$, and $|3 - 2x| = 2x - 3$ if $x > \frac{3}{2}$.

(b) $3 - 2x = 7$ gives $x = -2$; $3 - 2x = -7$ gives $x = 5$. Check: $|3 + 4| = 7$ and $|3 - 10| = 7$.

17 Converting temperatures



(a) Set $F = \frac{9}{5}C + 32$. Then $C = \frac{5}{9}(F - 32)$, so $F^{-1}(x) = \frac{5}{9}(x - 32)$.

(b) $F^{-1}(98.6) = \frac{5}{9}(66.6) = 37$, so $98.6^\circ\text{F} = 37^\circ\text{C}$.

(c) F^{-1} converts a Fahrenheit reading back to Celsius.

18 Domain traps with a root



(a) Take $g(x) = x^2 - 9$ (inner) and $f(x) = \sqrt{x}$ (outer).

(b) We need $x^2 - 9 \geq 0$, so $|x| \geq 3$: domain $(-\infty, -3] \cup [3, \infty)$.

(c) $(g \circ f)(x) = (\sqrt{x})^2 - 9 = x - 9$, but x must be in the domain of f , so the domain is $[0, \infty)$. The formula $x - 9$ alone would suggest all real numbers: the inner function restricts the domain.

19 Both orders, both domains



$(f \circ g)(x) = \sqrt{4 - x^2}$. We need $4 - x^2 \geq 0$, so $-2 \leq x \leq 2$: domain $[-2, 2]$.

$(g \circ f)(x) = (\sqrt{4 - x})^2 = 4 - x$. The inner function needs $4 - x \geq 0$, so $x \leq 4$: domain $(-\infty, 4]$.

20 Inverse after restriction

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- (a) $f(x) = (x + 3)^2 - 8$. For $x \geq -3$ the range is $[-8, \infty)$.
- (b) $y = (x + 3)^2 - 8$ with $x + 3 \geq 0$ gives $x = -3 + \sqrt{y + 8}$. So $f^{-1}(x) = -3 + \sqrt{x + 8}$, with domain $[-8, \infty)$ and range $[-3, \infty)$.
- (c) $f(-1) = 1 - 6 + 1 = -4$. And $f^{-1}(-4) = -3 + \sqrt{4} = -1$.
- (d) Then $x + 3 \leq 0$, so $x = -3 - \sqrt{y + 8}$ and $f^{-1}(x) = -3 - \sqrt{x + 8}$.

21 Inverse of a piecewise function

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- (a) For $x \leq 0$, $f(x) = x + 4 \leq 4$, range $(-\infty, 4]$. For $x > 0$, $f(x) = 2x + 6 > 6$, range $(6, \infty)$. Each piece is increasing and the ranges do not overlap, so f is one-to-one.
- (b) From $y = x + 4$: $x = y - 4$. From $y = 2x + 6$: $x = \frac{y - 6}{2}$. So $f^{-1}(x) = x - 4$ if $x \leq 4$, and $f^{-1}(x) = \frac{x - 6}{2}$ if $x > 6$. The domain is $(-\infty, 4] \cup (6, \infty)$.
- (c) $f(-3) = 1$ and $f^{-1}(1) = -3$. $f(2) = 10$ and $f^{-1}(10) = \frac{4}{2} = 2$.

22 Solving with composition

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- (a) $(f \circ g)(x) = (2x + 3)^2 - 1 = 24$ gives $(2x + 3)^2 = 25$, so $2x + 3 = 5$ or $2x + 3 = -5$: $x = 1$ or $x = -4$. Check: $g(1) = 5$, $f(5) = 24$; $g(-4) = -5$, $f(-5) = 24$.
- (b) $(g \circ f)(x) = 2(x^2 - 1) + 3 = 2x^2 + 1$. Then $2x^2 + 1 = x$ gives $2x^2 - x + 1 = 0$. The discriminant is $1 - 8 = -7 < 0$, so there is no real solution.

23 Moving points under a transformation

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- (a) A point (a, b) goes to $(a + 1, -2b + 3)$: $(-1, -3)$, $(1, 5)$, $(5, -7)$.
- (b) A point (a, b) goes to $(-a, b/2)$: $(2, 1.5)$, $(0, -0.5)$, $(-4, 2.5)$.
- Check one: $-2(3) + 3 = -3$, $-2(-1) + 3 = 5$, $-2(5) + 3 = -7$.

24

Parity of composites

(a) $(f \circ g)(-x) = f(g(-x)) = f(g(x)) = (f \circ g)(x)$ because g is even. So $f \circ g$ is even.

(b) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = -f(g(x))$ because g is odd and then f is odd. So $f \circ g$ is odd.

Example: $(f \circ g)(x) = (x^2 + 1)^3$. At $x = 2$ and $x = -2$ both give $5^3 = 125$.

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