

Factors and Multiples: math lesson, Grade 6 – download the PDF

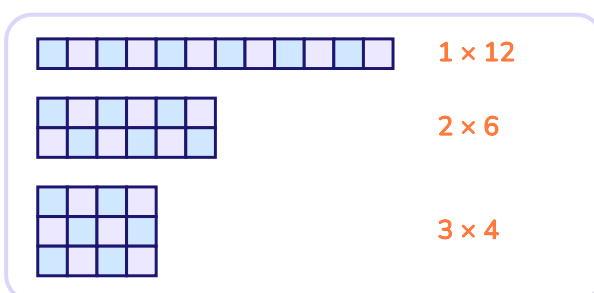
Two street lights blink at different speeds, a teacher splits a pile of supplies into identical kits, a band lines up in equal rows. Behind each of these situations hide **factors** and **multiples**. In this chapter you will learn to spot prime and composite numbers, test divisibility in seconds, and find the **greatest common factor (GCF)** and the **least common multiple (LCM)** to solve real word problems.

1. Factors and multiples

D Factor and multiple

A **factor** of a whole number n is a whole number that divides n with no remainder. A **multiple** of n is the product of n and any whole number $1, 2, 3, \dots$

Since $4 \times 6 = 24$, the numbers **4** and **6** are factors of **24**, and **24** is a multiple of both **4** and **6**. A number always has a finite list of factors, but its list of multiples never ends.



The picture shows why the factors of **12** come in pairs: **12** squares can be arranged as **1×12** , **2×6** or **3×4** . So the factors of **12** are **1, 2, 3, 4, 6** and **12**.

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Method: list the factors in pairs

1. Start with **$1 \times n$** .
2. Test **2, 3, 4, ...** in order. When a number divides **n** , write the pair.
3. Stop when the second number of a pair is smaller than the first one (or equal to it).
4. Write all the numbers of the pairs from smallest to largest.

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Example 1: factors of 36

Pairs: **1×36 , 2×18 , 3×12 , 4×9 , 6×6** . We stop at **6×6** . The factors of **36** are **1, 2, 3, 4, 6, 9, 12, 18, 36**.



Watch out

A factor is never larger than the number itself, and a multiple is never smaller than the number. Do not mix up the two words: the factors of **10** are **1, 2, 5, 10**, but its multiples are **10, 20, 30, ...**

2. Prime and composite numbers

Prime and composite

A **prime number** has exactly two factors: **1** and itself. A **composite number** has more than two factors. The number **1** is neither prime nor composite.

The prime numbers below **50** are **2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47**. Notice that **2** is the only even prime number: every other even number has **2** as an extra factor.

Zyro's tip

To test whether a number is prime, divide it by the primes **2, 3, 5, 7, ...** in order. You can stop as soon as the square of the prime you are testing is bigger than your number. For **97**, testing **2, 3, 5, 7** is enough because **$11 \times 11 = 121$** is already too big.

Watch out

Some composite numbers look prime. **51** seems like it should be prime, but **$51 = 3 \times 17$** . Always test the small primes before you decide.

3. Divisibility rules

A number is **divisible** by another when the division leaves no remainder. These rules let you check it without dividing.

Divisible by	Rule
2	The last digit is 0, 2, 4, 6 or 8.
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is divisible by both 2 and 3.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

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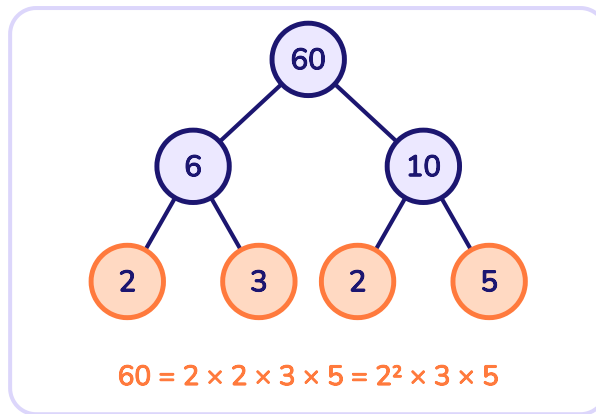
Example 2: testing 4,572

Last digit **2**: even, so divisible by **2**. Digit sum: $4 + 5 + 7 + 2 = 18$, which is divisible by **3** and by **9**. Last two digits: $72 = 4 \times 18$, so divisible by **4**. Being divisible by **2** and **3**, it is divisible by **6** too. It does not end in **0** or **5**, so it is not divisible by **5** or **10**.

Indeed, $4,572 = 36 \times 127$.

4. Prime factorization

Every composite number can be written as a product of prime numbers, and this product is **unique** (apart from the order). A **factor tree** helps you find it: split the number into two factors, then keep splitting until every branch ends on a prime.

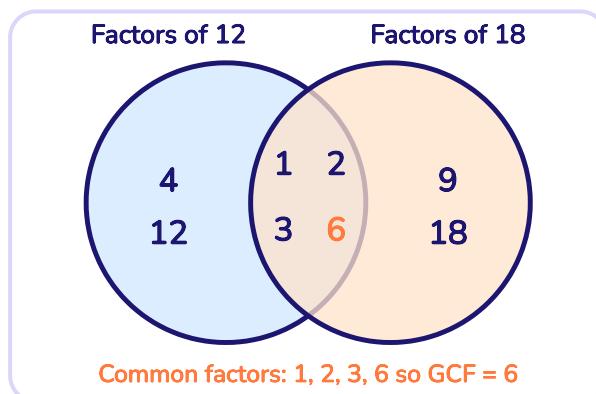


Here $60 = 6 \times 10 = (2 \times 3) \times (2 \times 5)$, so $60 = 2 \times 2 \times 3 \times 5$. With exponents, we write $60 = 2^2 \times 3 \times 5$. Whatever first split you choose (4×15 , 3×20 , ...), you always end with the same primes.

5. Greatest common factor (GCF)

D Greatest common factor

The **GCF** of two numbers is the largest number that is a factor of both.



The factors of **12** are **1, 2, 3, 4, 6, 12** and the factors of **18** are **1, 2, 3, 6, 9, 18**. The overlap of the diagram holds the common factors, and the largest is **6**. So $\text{GCF}(12, 18) = 6$.

M**Method: GCF with prime factorizations**

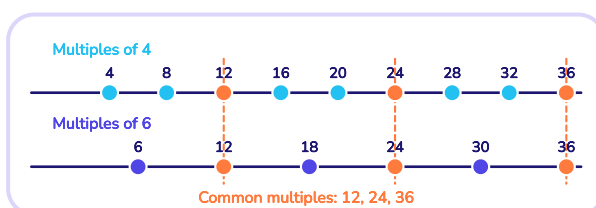
1. Write the prime factorization of each number.
2. Keep only the primes that appear in **both**, each with its **smaller** exponent.
3. Multiply them.

E**Example 3: GCF of 48 and 72**

$48 = 2^4 \times 3$ and $72 = 2^3 \times 3^2$. The smaller exponents are 2^3 and 3^1 , so $\text{GCF}(48, 72) = 2^3 \times 3 = 24$.

6. Least common multiple (LCM)**D****Least common multiple**

The **LCM** of two numbers is the smallest positive number that is a multiple of both.



The multiples of **4** are **4, 8, 12, 16, ...** and the multiples of **6** are **6, 12, 18, ...**. The first number on both lines is **12**, so $\text{LCM}(4, 6) = 12$. The next common multiples, **24** and **36**, are multiples of the LCM.

M**Method: LCM with prime factorizations**

1. Write the prime factorization of each number.
2. Keep every prime that appears in **either** number, each with its **larger** exponent.
3. Multiply them.

E**Example 4: LCM of 12 and 18**

$12 = 2^2 \times 3$ and $18 = 2 \times 3^2$. The larger exponents are 2^2 and 3^2 , so $\text{LCM}(12, 18) = 4 \times 9 = 36$.

P**A handy check**

For two numbers a and b : $\text{GCF}(a, b) \times \text{LCM}(a, b) = a \times b$. For 12 and 18:
 $6 \times 36 = 216 = 12 \times 18$.

7. The distributive property with the GCF

The distributive property says $a(b + c) = ab + ac$. Read from right to left, it lets you **factor out** the GCF of a sum.

E**Example 5: rewriting $36 + 48$**

$\text{GCF}(36, 48) = 12$. Since $36 = 12 \times 3$ and $48 = 12 \times 4$, we get
 $36 + 48 = 12(3 + 4) = 12 \times 7 = 84$.

The numbers left inside the parentheses, **3** and **4**, have no common factor other than **1**. If they did, the number you factored out was not the greatest one.



Watch out

Do not forget the **1** when a term equals the common factor. For example,
 $15 + 45 = 15(1 + 3)$, not $15(3)$.

8. Word problems with the GCF and the LCM

Ask yourself one question to choose the right tool.

- Are you **splitting** things into equal groups, equal pieces or equal rows, as large as possible? Use the **GCF**.
- Are two events **repeating**, and you want to know when they happen together again, or the smallest amount that fits both patterns? Use the **LCM**.



Example 6: GCF

A coach has **24** apples and **36** oranges and wants to make identical fruit bags with nothing left over. The most bags is $\text{GCF}(24, 36) = 12$. Each bag holds $24 \div 12 = 2$ apples and $36 \div 12 = 3$ oranges.



Example 7: LCM

Two buses leave the station together at 9:00 a.m. One leaves every **8** minutes, the other every **12** minutes. They leave together again after $\text{LCM}(8, 12) = 24$ minutes, at 9:24 a.m.

Key takeaways

- A factor divides a number exactly; a multiple is the number times **1, 2, 3, ...**
- A prime number has exactly two factors; **1** is neither prime nor composite; **2** is the only even prime.
- Use the divisibility rules for **2, 3, 4, 5, 6, 9** and **10** before dividing.
- Prime factorization is unique: **$60 = 2^2 \times 3 \times 5$** .
- GCF: common primes, **smaller** exponents. LCM: all primes, **larger** exponents.
- **$\text{GCF}(a, b) \times \text{LCM}(a, b) = a \times b$** .
- Factor out the GCF to rewrite a sum: **$36 + 48 = 12(3 + 4)$** .
- Splitting into equal groups means GCF; repeating events means LCM.



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