

Factoring Polynomials: math lesson, Grade 9 – download the PDF

Factoring is multiplication run backward: instead of expanding a product, you rewrite a polynomial as a product of simpler pieces. This skill unlocks quadratic equations, because a product equals zero only when one of its factors does. In this chapter you will build a complete toolkit: GCF, grouping, trinomials, special patterns and the zero product property.

1. Greatest common factor (GCF)

D Definition

The **greatest common factor** of the terms of a polynomial is the largest monomial that divides every term. To **factor out** the GCF, write it in front of parentheses and put the leftover quotients inside.

Always look for a GCF first, even when other methods will follow. Take the GCF of the coefficients (largest whole number dividing all of them), then each variable with its *smallest* exponent.

E**Example 1**

Factor $18x^3 - 24x^2 + 12x$. The coefficients 18, 24 and 12 share the factor 6, and every term contains at least one x . The GCF is $6x$.

$$18x^3 - 24x^2 + 12x = 6x(3x^2 - 4x + 2)$$

Check by expanding: $6x \cdot 3x^2 = 18x^3$, $6x \cdot (-4x) = -24x^2$, $6x \cdot 2 = 12x$.

**Common mistake**

If a term equals the GCF, a **1** must remain: $5x^2 + 5x = 5x(x + 1)$, not $5x(x)$.
Always expand to check.

2. Factoring by grouping

When a polynomial has four terms, split it into two pairs, factor the GCF out of each pair, and hope that the same binomial shows up twice. Then factor that binomial out.

M**Method**

1. Group the terms in two pairs.
2. Factor the GCF out of each pair.
3. If both pairs now contain the same binomial, factor it out as a common factor.

E**Example 2**

Factor $x^3 + 4x^2 + 3x + 12$.

$$x^2(x + 4) + 3(x + 4) = (x + 4)(x^2 + 3)$$

Expanding $(x + 4)(x^2 + 3)$ gives $x^3 + 3x + 4x^2 + 12$, which matches the original polynomial.

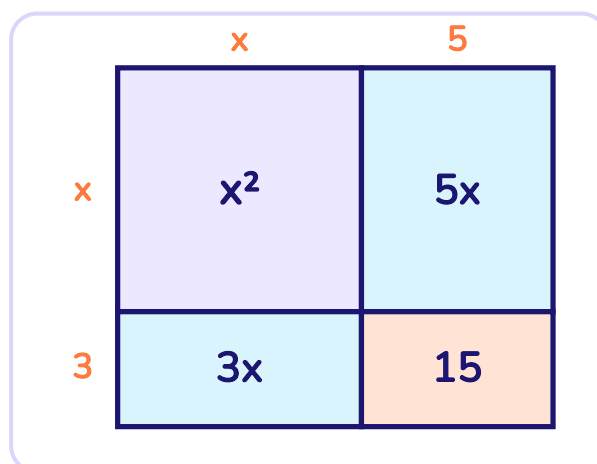
If the signs of the second pair make the binomials opposite, factor out a negative GCF, for example $-6x - 15$ becomes $-3(2x + 5)$.

3. Trinomials $x^2 + bx + c$

A trinomial with leading coefficient 1 factors as $(x + p)(x + q)$ where the two numbers satisfy

P**Property**

$x^2 + bx + c = (x + p)(x + q)$ exactly when $p \cdot q = c$ and $p + q = b$.



The area model above shows why: the rectangle of sides $x + 3$ and $x + 5$ is cut into pieces of area x^2 , $5x$, $3x$ and 15 . The middle terms add up: $5x + 3x = 8x$, so

$$(x + 3)(x + 5) = x^2 + 8x + 15.$$

E Example 3

Factor $x^2 - 2x - 35$. We need two numbers with product -35 and sum -2 . The pair -7 and 5 works.

$$x^2 - 2x - 35 = (x - 7)(x + 5)$$

When c is positive, p and q share the sign of b . When c is negative, they have opposite signs and the one with the larger absolute value takes the sign of b .

I 4. Trinomials $ax^2 + bx + c$

M Method: the ac method

1. Multiply $a \cdot c$.
2. Find two numbers with product ac and sum b .
3. Rewrite the middle term as the sum of two terms using those numbers.
4. Factor by grouping.

E Example 4

Factor $6x^2 + 11x + 3$. Here $ac = 18$ and we need a sum of 11, so the numbers are 9 and 2.

$$6x^2 + 9x + 2x + 3 = 3x(2x + 3) + 1(2x + 3) = (2x + 3)(3x + 1)$$

E

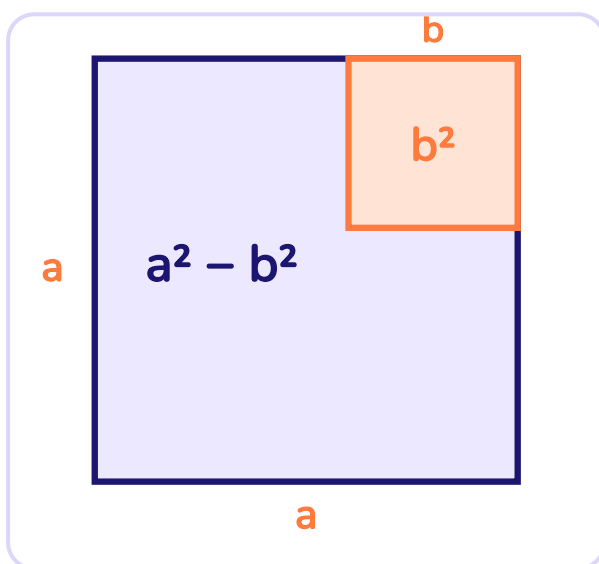
Example 5

Factor $3x^2 - 10x - 8$. Now $ac = -24$ and the numbers with sum -10 are -12 and 2 .

$$3x^2 - 12x + 2x - 8 = 3x(x - 4) + 2(x - 4) = (x - 4)(3x + 2)$$

If no pair of integers works, the trinomial is called *prime* over the integers.

5. Special patterns



P

Difference of two squares

$a^2 - b^2 = (a - b)(a + b)$. The picture shows the idea: remove a square of side b from a square of side a and the remaining area is $a^2 - b^2$. A *sum* of squares such as $a^2 + b^2$ does not factor over the integers.

P**Perfect square trinomials**

$$a^2 + 2ab + b^2 = (a + b)^2 \text{ and } a^2 - 2ab + b^2 = (a - b)^2.$$

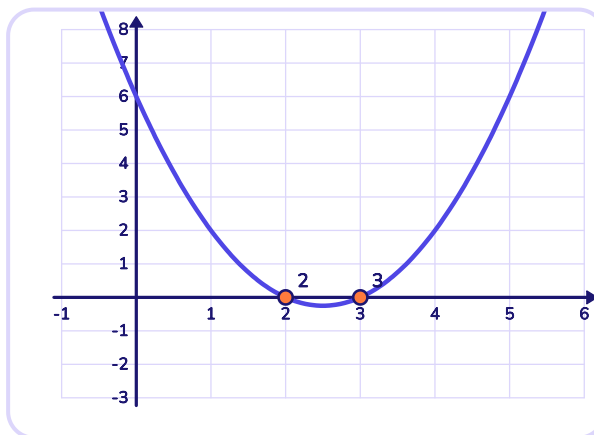
E**Example 6**

$$49x^2 - 64 = (7x)^2 - 8^2 = (7x - 8)(7x + 8).$$

$25x^2 - 30x + 9$: the first and last terms are $(5x)^2$ and 3^2 , and the middle term is $2 \cdot 5x \cdot 3 = 30x$. So $25x^2 - 30x + 9 = (5x - 3)^2$.

!**Zyro's tip**

On my planet we say: factor out the GCF first, then look for a pattern. After that, count the terms: 2 terms (difference of squares?), 3 terms (trinomial?), 4 terms (grouping?).

6. The zero product property

P Zero product property

If $A \cdot B = 0$, then $A = 0$ or $B = 0$ (or both). This works for any number of factors, but only when the product equals *zero*.

The parabola above is $y = (x - 2)(x - 3)$. It crosses the x-axis where a factor is zero, at $x = 2$ and $x = 3$. Factoring therefore tells you where a graph meets the x-axis.

I 7. Solving equations by factoring

M Method

1. Move every term to one side so the other side is 0.
2. Factor the polynomial completely.
3. Set each factor equal to 0 and solve.
4. Check each solution in the original equation.

E Example 7

Solve $x^2 + 6x = 16$.

$$x^2 + 6x - 16 = 0 \Rightarrow (x + 8)(x - 2) = 0 \Rightarrow x = -8 \text{ or } x = 2$$

Check: $(-8)^2 + 6(-8) = 64 - 48 = 16$ and $2^2 + 6 \cdot 2 = 16$.



Common mistake

Never divide both sides by x to simplify $x^2 = 7x$: you lose the solution $x = 0$. Write $x^2 - 7x = 0$, then $x(x - 7) = 0$. Also, $(x - 2)(x + 3) = 14$ does *not* mean $x - 2 = 14$; the property only applies to a product of 0.

Key takeaways

- Always factor out the greatest common factor first, and check by expanding.
- Four terms: try grouping. Three terms: find two numbers with product c (or ac) and sum b .
- $a^2 - b^2 = (a - b)(a + b)$; $a^2 \pm 2ab + b^2 = (a \pm b)^2$.
- Zero product property: $AB = 0$ means $A = 0$ or $B = 0$.
- To solve by factoring, set one side to 0, factor, set each factor to 0, then check.



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