

Exponents and Scientific Notation: math lesson, Grade 8 – download the PDF

A single grain of rice is tiny, yet a chessboard story says that doubling one grain on each square would pile up a mountain of rice. Powers are the shortcut that makes such huge (and microscopic) numbers easy to write and compare. In this chapter you will master the rules of exponents, extend them to zero and negative exponents, and use **scientific notation** to handle the biggest and smallest quantities in science.

1. What is an exponent?

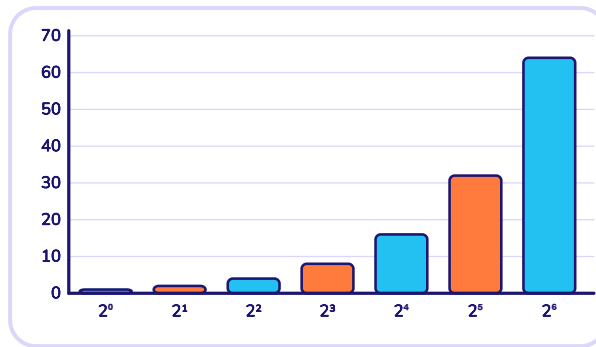
D Power, base and exponent

For a number a and a positive integer n , the power a^n means a multiplied by itself n times:

$$a^n = \underbrace{a \times a \times \cdots \times a}_{n \text{ factors}}$$

The number a is the **base** and n is the **exponent**. We also say that a^n is “ a to the n th power”.

For example, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ and $10^3 = 10 \times 10 \times 10 = 1,000$. By convention, $a^1 = a$. The exponent only counts the factors: 2^5 is **not** 2×5 .



The chart shows how quickly powers grow: each time the exponent goes up by 1, the value of 2^n doubles. Powers of 10 are even more dramatic, and they are the key to scientific notation later in this chapter.



Watch the sign

The exponent applies only to what is directly in front of it. $(-3)^2 = (-3)(-3) = 9$, but $-3^2 = -(3 \times 3) = -9$. An odd exponent keeps a negative base negative: $(-2)^3 = -8$.

2. The product rule

What happens when you multiply two powers with the same base? Write them out: $3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3) = 3^6$. There are $2 + 4 = 6$ factors in total.



Product rule

For any nonzero base a and integers m and n :

$$a^m \times a^n = a^{m+n}$$

When multiplying powers with the same base, **keep the base and add the exponents**.

E**Example 1**

Simplify $5^3 \times 5^4$ and $(4x^2)(3x^5)$.

$$5^3 \times 5^4 = 5^{3+4} = 5^7.$$

Multiply the numbers, then use the rule on the variable:

$$(4x^2)(3x^5) = 12x^{2+5} = 12x^7.$$

**Same base only**

The rule works only when the bases match. $2^3 \times 3^2 = 8 \times 9 = 72$ cannot be written as one power of 6. And never multiply the bases: $2^3 \times 2^4 = 2^7$, not 4^7 .

3. The quotient rule

Dividing powers cancels common factors. For instance,

$$\frac{7^5}{7^2} = \frac{7 \times 7 \times 7 \times 7 \times 7}{7 \times 7} = 7 \times 7 \times 7 = 7^3.$$

P**Quotient rule**

For any nonzero base a and integers m and n :

$$\frac{a^m}{a^n} = a^{m-n}$$

When dividing powers with the same base, **keep the base and subtract the exponents**.

E**Example 2**

Simplify $\frac{10^9}{10^4}$ and $\frac{12a^8}{4a^3}$.

$$\frac{10^9}{10^4} = 10^{9-4} = 10^5 = 100,000.$$

Divide the numbers and use the rule on the variable: $\frac{12a^8}{4a^3} = 3a^{8-3} = 3a^5$.

4. Power of a power

Now raise a power to another power: $(2^3)^2 = 2^3 \times 2^3 = 2^{3+3} = 2^6 = 64$. Two groups of three factors make six factors.

P**Power of a power, product and quotient**

For nonzero a and b and integers m and n :

$$(a^m)^n = a^{m \times n} \quad (ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

When raising a power to a power, **multiply the exponents**. An exponent outside parentheses applies to every factor inside.

E**Example 3**

Simplify $(x^4)^5$ and $(3x^2)^3$.

$$(x^4)^5 = x^{4 \times 5} = x^{20}.$$

$$(3x^2)^3 = 3^3 \times (x^2)^3 = 27x^6. \text{ The exponent 3 hits both the 3 and the } x^2.$$

**Add, multiply, or neither?**

Compare $(2^3)^2 = 2^6 = 64$ with $2^3 \times 2^2 = 2^5 = 32$. Powers multiplied together: add exponents. Power raised to a power: multiply exponents.

5. Zero and negative exponents

What should 2^0 or 2^{-3} mean? Look for a pattern. Each time the exponent drops by 1, the value is divided by 2:

Power	Value
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$

To keep the pattern (and the rules above) working, we define the following.

D Zero and negative exponents

For any nonzero number a and positive integer n :

$$a^0 = 1 \qquad a^{-n} = \frac{1}{a^n}$$

A negative exponent means “take the reciprocal”. It does not make the number negative.

E**Example 4**

Evaluate 5^{-2} and $\left(\frac{2}{3}\right)^{-2}$, then simplify $x^3 \times x^{-5}$.

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}.$$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}, \text{ because a negative exponent flips the fraction.}$$

$$x^3 \times x^{-5} = x^{3+(-5)} = x^{-2} = \frac{1}{x^2}.$$

!**Zyro's trick**

On my planet we say: “negative exponent, flip it upstairs or downstairs.” Move the power to the other side of the fraction bar and its exponent turns positive!

⚠**Common mistake**

2^{-3} equals $\frac{1}{8}$, not -8 . Also remember $a^0 = 1$, never 0 (for $a \neq 0$).

6. Scientific notation

Scientists deal with numbers like the number of seconds in a century or the width of a virus. Writing all those zeros is slow and error-prone, so we use powers of 10.

D**Scientific notation**

A number is in scientific notation when it is written as

$$a \times 10^n \quad \text{with } 1 \leq |a| < 10 \text{ and } n \text{ an integer.}$$

The factor ***a*** has exactly one nonzero digit to the left of the decimal point.



move the decimal point 7 places to the left

$$45,300,000 = 4.53 \times 10^7$$

M**Method: converting to scientific notation**

1. Place the decimal point after the first nonzero digit to get ***a***.
2. Count how many places the decimal point moved.
3. If you moved it to the **left** (large number), the exponent is positive. If you moved it to the **right** (small number), the exponent is negative.

E**Example 5**

Write **45,300,000** and **0.00062** in scientific notation, then write **3.8×10^5** in standard form.

$$45,300,000 = 4.53 \times 10^7 \text{ (7 places to the left).}$$

$$0.00062 = 6.2 \times 10^{-4} \text{ (4 places to the right).}$$

$$3.8 \times 10^5 = 380,000 \text{ (move the decimal point 5 places to the right).}$$

**Not quite scientific**

45×10^6 and **0.8×10^3** are correct values but not in scientific notation, because ***a*** is not between 1 and 10. Rewrite them as **4.5×10^7** and **8×10^2** .

7. Operations with scientific notation

Multiplying and dividing. Group the decimal parts and the powers of 10 separately, use the product or quotient rule on the powers of 10, and finally fix the decimal part if it leaves the range from 1 to 10.

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Example 6

Compute $(3 \times 10^4)(5 \times 10^6)$ and $\frac{8.4 \times 10^9}{2.1 \times 10^4}$.

$$(3 \times 10^4)(5 \times 10^6) = 15 \times 10^{10} = 1.5 \times 10^{11}.$$

$$\frac{8.4 \times 10^9}{2.1 \times 10^4} = \frac{8.4}{2.1} \times 10^{9-4} = 4 \times 10^5.$$

Adding and subtracting. You can only add or subtract terms that carry the **same power of 10**. If the powers differ, rewrite one number first.

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Example 7

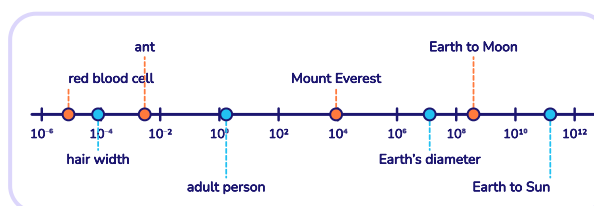
Compute $3.2 \times 10^5 + 4.1 \times 10^5$ and $6 \times 10^4 + 5 \times 10^3$.

Same power: $3.2 \times 10^5 + 4.1 \times 10^5 = (3.2 + 4.1) \times 10^5 = 7.3 \times 10^5$.

Different powers: $5 \times 10^3 = 0.5 \times 10^4$, so $6 \times 10^4 + 0.5 \times 10^4 = 6.5 \times 10^4$.

8. Comparing very large and small quantities

Scientific notation makes comparisons easy. For positive numbers, first compare the exponents: the larger exponent wins. If the exponents are equal, compare the decimal parts *a*. To find *how many times* larger one quantity is than another, divide them.



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Example 8

The Moon is about 3.84×10^8 m from Earth, and the Sun is about 1.5×10^{11} m away. Which is farther, and by what factor?

Since $11 > 8$, the Sun is farther. Divide:

$$\frac{1.5 \times 10^{11}}{3.84 \times 10^8} = \frac{1.5}{3.84} \times 10^3 \approx 0.39 \times 10^3 \approx 3.9 \times 10^2.$$

The Sun is about 390 times as far away as the Moon. For U.S. units, the Moon distance is about 2.39×10^5 miles, using $1 \text{ mile} \approx 1,609 \text{ m}$.

Negative exponents work the same way: 7×10^{-5} m (a hair's width) is ten times larger than 7×10^{-6} m (a red blood cell), because the exponents differ by 1. Careful: with negative exponents, 10^{-3} is **larger** than 10^{-5} , since $0.001 > 0.00001$.

Key takeaways

- a^n means a multiplied by itself n times; a is the base and n the exponent.
- **Product rule:** $a^m \times a^n = a^{m+n}$. **Quotient rule:** $\frac{a^m}{a^n} = a^{m-n}$. Same base only.
- **Power of a power:** $(a^m)^n = a^{mn}$, and $(ab)^n = a^n b^n$.
- $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ for $a \neq 0$; a negative exponent flips the base, it does not make the value negative.
- Scientific notation: $a \times 10^n$ with $1 \leq |a| < 10$; positive n for big numbers, negative n for tiny ones.
- To add or subtract, match the powers of 10 first; to multiply or divide, handle the powers of 10 with the exponent rules.
- To compare, look at the exponents first, then at the decimal parts; divide to get “how many times”.



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