

Exponent Rules and Polynomials: math lesson, Grade 9 – download the PDF

Population counts, computer storage, the distance to a star: huge and tiny numbers are everywhere, and exponents let you write them in a flash. In this chapter you will master the **rules of exponents**, learn to write numbers in **scientific notation**, and then move on to **polynomials**: how to name them, add them, subtract them and multiply them quickly.

1. Product and quotient rules

D Exponent

In a^n , the base a is multiplied by itself n times. For example $x^4 = x \cdot x \cdot x \cdot x$. The small number n is the **exponent**.

P**Product and quotient rules**

For any nonzero base a and whole numbers m and n :

$$a^m \cdot a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

Same base: you **add** the exponents when multiplying and **subtract** them when dividing.

Why does it work? $x^3 \cdot x^2 = (x \cdot x \cdot x)(x \cdot x) = x^5$: five factors of x in total.

E**Example 1**

Simplify $3x^2 \cdot 4x^6$ and $\frac{15y^9}{3y^4}$.

Multiply the coefficients, add the exponents: $3x^2 \cdot 4x^6 = 12x^{2+6} = 12x^8$.

Divide the coefficients, subtract the exponents: $\frac{15y^9}{3y^4} = 5y^{9-4} = 5y^5$.

⚠**Common mistake**

The rule needs the **same base**. $x^3 \cdot y^2$ cannot be simplified, and $2^3 \cdot 2^4 = 2^7$, not 4^7 : the bases are **never** multiplied.

2. Power rule and zero exponent

P Power rules

$$(a^m)^n = a^{mn} \quad (ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

A power of a power: **multiply** the exponents. A power of a product: every factor gets the exponent.

P Zero exponent

For every nonzero number a , $a^0 = 1$.

Look at the pattern: $\frac{x^5}{x^5} = 1$ because anything divided by itself is 1, and the quotient rule gives $x^{5-5} = x^0$. So x^0 *must* equal 1.

E Example 2

Simplify $(2x^3)^4$ and $(-3a^2b)^3$.

$$(2x^3)^4 = 2^4 \cdot (x^3)^4 = 16x^{12}.$$

$$(-3a^2b)^3 = (-3)^3 \cdot a^6 \cdot b^3 = -27a^6b^3.$$



Watch the coefficient

$(2x)^3 = 8x^3$, not $2x^3$: the exponent applies to the 2 as well.

3. Negative exponents



Negative exponent

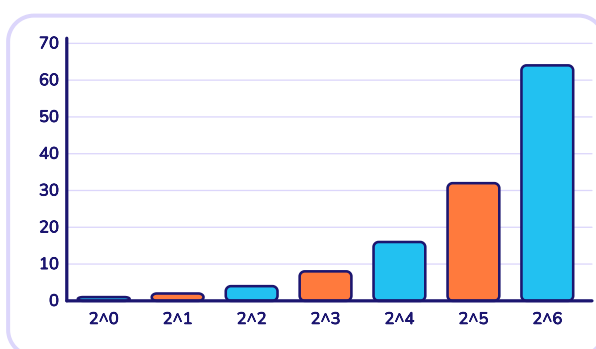
For $a \neq 0$ and a positive whole number n :

$$a^{-n} = \frac{1}{a^n}$$

A negative exponent does **not** make the number negative; it flips the base to the other side of the fraction bar.

n	3	2	1	0	-1	-2	-3
2^n	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

In the table, each step to the right divides by 2. Continuing past $2^0 = 1$ leads naturally to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.



E**Example 3**

Rewrite with positive exponents: 5^{-2} , $\frac{x^3}{x^7}$, and $4x^{-2}y^3$.

$$5^{-2} = \frac{1}{5^2} = \frac{1}{25}.$$

$$\frac{x^3}{x^7} = x^{3-7} = x^{-4} = \frac{1}{x^4}.$$

$$4x^{-2}y^3 = \frac{4y^3}{x^2}: \text{ only } x \text{ carries the negative exponent, so only } x \text{ moves.}$$

4. Scientific notation**D****Scientific notation**

A number is in scientific notation when it is written

$$a \times 10^n \quad \text{with } 1 \leq a < 10 \text{ and } n \text{ an integer.}$$

M**Converting**

1. Place the decimal point after the first nonzero digit to get a .
2. Count how many places the point moved: that is $|n|$.
3. Large number: n is positive. Number between 0 and 1: n is negative.

E**Example 4**

Write 45,000,000 and 0.00032 in scientific notation.

$$45,000,000 = 4.5 \times 10^7 \text{ (the point moved 7 places left).}$$

$$0.00032 = 3.2 \times 10^{-4} \text{ (the point moved 4 places right).}$$

To multiply or divide, handle the decimals and the powers of 10 separately, then fix the result if ***a*** leaves the interval from 1 to 10.

E**Example 5**

Compute $(8 \times 10^4)(5 \times 10^3)$ and $\frac{6 \times 10^8}{1.5 \times 10^3}$.

$$(8 \times 10^4)(5 \times 10^3) = 40 \times 10^7 = 4 \times 10^8.$$

$$\frac{6 \times 10^8}{1.5 \times 10^3} = 4 \times 10^5.$$

5. Classifying polynomials

D**Polynomial**

A **monomial** is a number, a variable, or a product of a number and variables with whole-number exponents, such as **$7x$** or **$-4x^3$** . A **polynomial** is a monomial or a sum of monomials, called its **terms**.

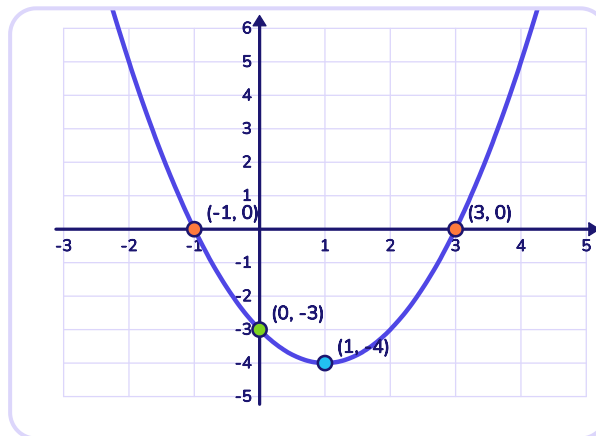
- The **degree of a term** is its exponent on the variable (a plain number has degree 0).
- The **degree of a polynomial** is the largest degree of its terms.

- In **standard form** the terms go from highest to lowest degree; the first coefficient is the **leading coefficient**.
- One term: **monomial**. Two terms: **binomial**. Three terms: **trinomial**.

E Example 6

Describe $P(x) = 4x - 7 + 2x^3$.

Standard form: $2x^3 + 4x - 7$. It has three terms, so it is a trinomial. Its degree is 3 and its leading coefficient is 2.



The graph above belongs to a degree 2 polynomial. A degree 2 polynomial draws a parabola, and the points where it meets the x-axis are the values that make the polynomial equal to 0.

I 6. Adding and subtracting polynomials

M Method

1. To subtract, change the sign of **every** term of the second polynomial.
2. Group the **like terms** (same variable, same exponent).
3. Add the coefficients; the exponents do not change.

E**Example 7**

Compute $(3x^2 + 5x - 4) + (2x^2 - 7x + 9)$ and $(6x^2 - x + 3) - (2x^2 + 4x - 5)$.

Sum: $5x^2 - 2x + 5$.

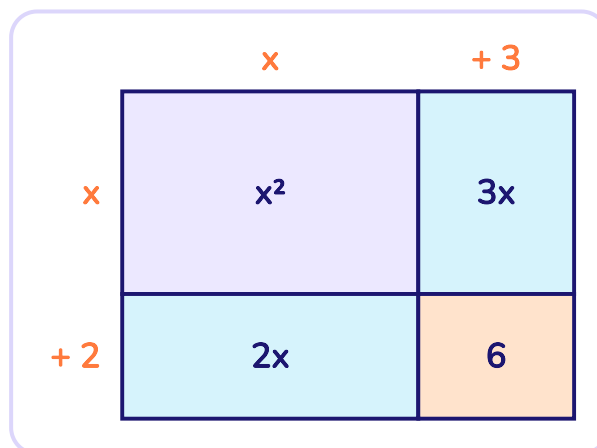
Difference: $6x^2 - x + 3 - 2x^2 - 4x + 5 = 4x^2 - 5x + 8$.

**The minus sign**

In $A - (x - 5)$, the sign of **both** terms flips: $-x + 5$, not $-x - 5$.

7. Multiplying polynomials

Multiplying uses the distributive property, and each product of terms follows the product rule for exponents.



The rectangle has sides $x + 3$ and $x + 2$. Its total area is the sum of four pieces:
 $(x + 3)(x + 2) = x^2 + 3x + 2x + 6 = x^2 + 5x + 6$. The pattern is called **FOIL** (First, Outer, Inner, Last) for two binomials.

E**Example 8**

Expand $3x(x^2 - 4x + 2)$ and $(2x - 3)(x^2 + 4x - 1)$.

$$3x(x^2 - 4x + 2) = 3x^3 - 12x^2 + 6x.$$

$$(2x - 3)(x^2 + 4x - 1) = 2x^3 + 8x^2 - 2x - 3x^2 - 12x + 3 = 2x^3 + 5x^2 - 14x + 3$$

Check at $x = 1$: $(-1)(4) = -4$ and $2 + 5 - 14 + 3 = -4$.

!**Zyro's tip**

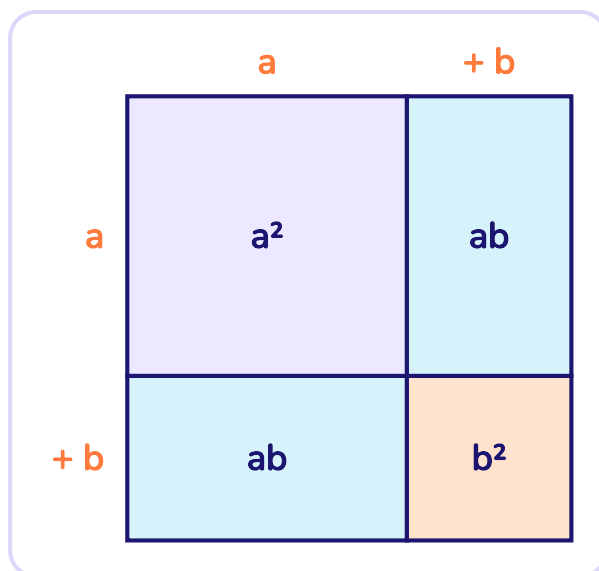
On my planet we test every expansion by plugging in a small number such as $x = 1$ or $x = 2$ in both the factored and the expanded form. If the two results differ, hunt for the error!

8. Special products**P****Three patterns worth memorizing**

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)(a - b) = a^2 - b^2$$



The square of side $a + b$ is cut into a^2 , two rectangles ab and b^2 . Adding them gives the first identity. In the last one, the middle terms $+ab$ and $-ab$ cancel.



Not $a^2 + b^2$

$(x + 6)^2 \neq x^2 + 36$. Do not forget the middle term $2ab$, here $12x$.



Example 9

Expand $(x + 6)^2$, $(3x - 5)^2$, $(4x + 7)(4x - 7)$; then compute 51^2 mentally.

$$(x + 6)^2 = x^2 + 12x + 36.$$

$$(3x - 5)^2 = 9x^2 - 30x + 25.$$

$$(4x + 7)(4x - 7) = 16x^2 - 49.$$

$$51^2 = (50 + 1)^2 = 2500 + 100 + 1 = 2601.$$

Key takeaways

- Same base: $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$, $(a^m)^n = a^{mn}$.
- $a^0 = 1$ for $a \neq 0$, and $a^{-n} = \frac{1}{a^n}$.
- Scientific notation: $a \times 10^n$ with $1 \leq a < 10$.
- A polynomial is named by its number of terms and its degree; write it in standard form.
- To add or subtract, combine like terms; flip every sign when subtracting.
- To multiply, distribute every term of one factor to every term of the other.
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$ and $(a + b)(a - b) = a^2 - b^2$.



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