

Exponential Functions: math lesson, Grade 11 – download the PDF

A viral video doubles its audience every day. A new car loses a slice of its value every year. A savings account earns interest on its own interest. All three stories share one pattern: the quantity is multiplied by the same factor again and again. In this chapter you will learn the functions that describe that pattern, graph them, shift them, and use them to model money, populations and decay.

1. What makes a function exponential?

D Exponential function

An exponential function has the form $f(x) = a \cdot b^x$, where $a \neq 0$ is the **initial value**, and $b > 0, b \neq 1$ is the **base** (also called the growth or decay factor). The variable x is in the **exponent**.

The base cannot be **1** (then $1^x = 1$ and the function is just a constant), and it cannot be negative (then b^x would fail to exist for many real numbers x , such as $x = \frac{1}{2}$).

E

Example 1: a table of values

Let $f(x) = 3 \cdot 2^x$. Then $f(0) = 3 \cdot 1 = 3$, $f(1) = 6$, $f(2) = 12$, $f(3) = 24$ and $f(-1) = 3 \cdot 2^{-1} = 1.5$.

x	-1	0	1	2	3
$f(x)$	1.5	3	6	12	24

Each time x goes up by 1, the output is **multiplied by 2**. That constant ratio is the signature of an exponential function; a linear function would add the same amount instead.



Watch out

x^2 and 2^x are very different. In x^2 the variable is the base (a power function); in 2^x the variable is the exponent. At $x = 5$, $x^2 = 25$ but $2^x = 32$, and the gap explodes after that.

2. Exponential growth and decay

P

Growth or decay?

For $f(x) = a \cdot b^x$ with $a > 0$:

If $b > 1$, the function shows **exponential growth**. If $0 < b < 1$, it shows **exponential decay**.

A change of r per period (written as a decimal) gives the factor $b = 1 + r$ for growth and $b = 1 - r$ for decay.

M**From a percent to a factor**

1. Write the percent as a decimal: 20% becomes **0.20**.
2. Add it for growth ($1 + 0.20 = 1.2$) or subtract it for decay ($1 - 0.20 = 0.8$).
3. Use that factor as the base and the starting amount as ***a***.

E**Example 2: growth**

A culture starts with 500 bacteria and grows 20% every hour. The model is

$P(t) = 500(1.2)^t$. After 5 hours: $P(5) = 500 \cdot 1.2^5 = 500 \cdot 2.48832 = 1244.16$, so about 1,244 bacteria.

E**Example 3: decay**

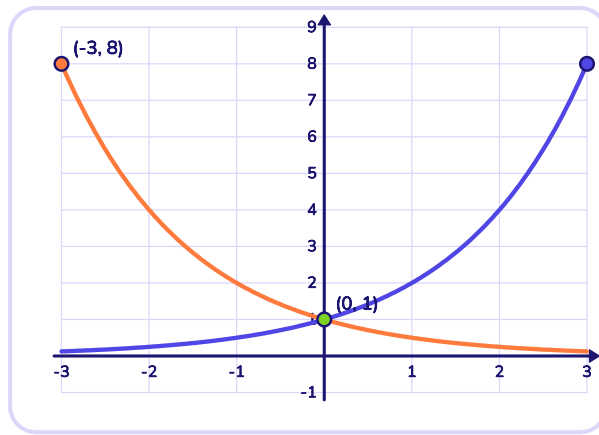
A van is worth 24,000 dollars and loses 15% of its value each year. The model is

$V(t) = 24000(0.85)^t$. After 4 years:

$V(4) = 24000 \cdot 0.85^4 = 24000 \cdot 0.52200625 = 12528.15$, so about 12,528 dollars.

3. Graphing exponential functions

Here are the graphs of $y = 2^x$ (growth) and $y = \left(\frac{1}{2}\right)^x$ (decay). They are mirror images of each other across the y-axis.



P Key features of $f(x) = a \cdot b^x$

- Domain: all real numbers. Range: $y > 0$ when $a > 0$.
- The y-intercept is $(0, a)$, because $b^0 = 1$.
- The x-axis, $y = 0$, is a **horizontal asymptote**: the curve gets closer and closer to it but never touches it.
- There is no x-intercept. The function is increasing if $b > 1$ and decreasing if $0 < b < 1$.

! Zyro's tip

To sketch fast, plot the y-intercept $(0, a)$, then the point at $x = 1$, which is $(1, ab)$. Draw the curve hugging the asymptote on one side and shooting up on the other.

4. Transformations of exponential functions

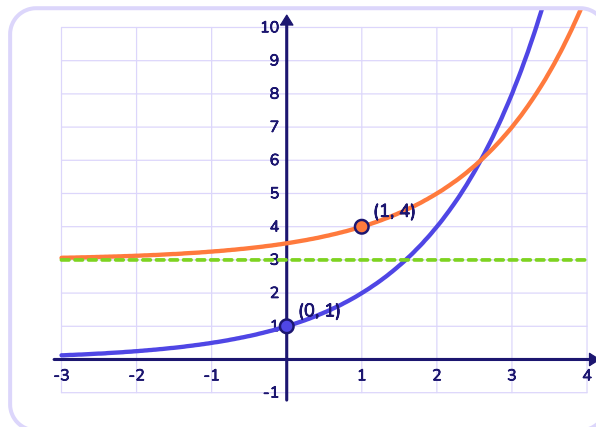
Exponentials move like any other function. For $g(x) = a \cdot b^{x-h} + k$:

- h shifts the graph **right** by h units (left if h is negative).
- k shifts the graph **up** by k units, and the asymptote moves to $y = k$.
- $a < 0$ reflects the graph across the x-axis; a larger $|a|$ stretches it vertically.

E

Example 4: shifting

Graph $g(x) = 2^{x-1} + 3$. Start from $y = 2^x$: move it 1 unit right and 3 units up. The point $(0, 1)$ goes to $(1, 4)$, the asymptote goes from $y = 0$ to $y = 3$, and the range becomes $y > 3$. The y -intercept is $g(0) = 2^{-1} + 3 = 3.5$.



5. The number e

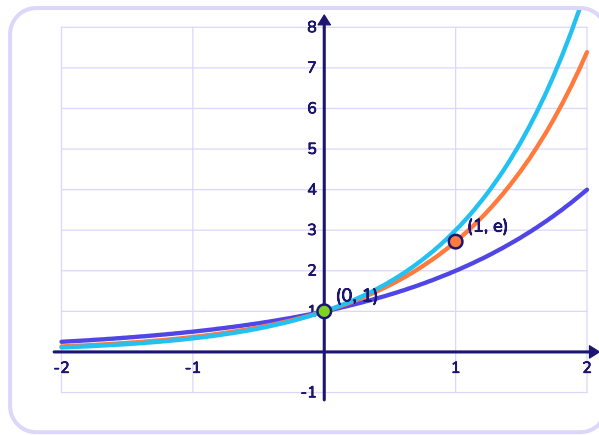
Suppose you invest 1 dollar at 100% interest per year. If the interest is added once, you end with 2 dollars. If it is added n times a year, you end with $\left(1 + \frac{1}{n}\right)^n$. More frequent additions help, but the total does not grow forever.

n	1	2	12	100	1,000	1,000,000
$\left(1 + \frac{1}{n}\right)^n$	2	2.25	2.6130	2.7048	2.7169	2.71828

D

The number e

The values settle toward an irrational number called $e \approx 2.71828$. The function $f(x) = e^x$ is the **natural exponential function**. Its graph lies between those of 2^x and 3^x , and it passes through $(0, 1)$ and $(1, e)$.



The inverse of e^x is the **natural logarithm**: if $e^t = c$, then $t = \ln c$. Your calculator has both keys, and you will need **ln** to solve for an unknown exponent.

6. Compound interest

P

Compound interest formula

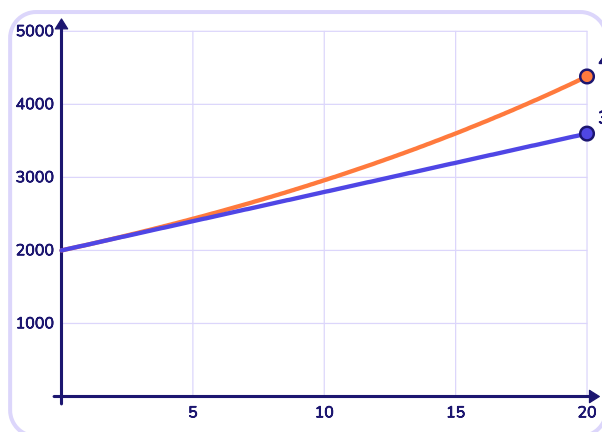
If P dollars earn an annual rate r (as a decimal), compounded n times per year for t years, the balance is

$$A = P \left(1 + \frac{r}{n} \right)^{nt}.$$

E

Example 5: quarterly compounding

Invest 2,000 dollars at 4% compounded quarterly for 6 years. Here $P = 2000$, $r = 0.04$, $n = 4$, $t = 6$, so $A = 2000(1 + 0.01)^{24} = 2000 \cdot 1.26973 \dots \approx 2539.47$. The balance is about 2,539.47 dollars. With annual compounding you would get $2000 \cdot 1.04^6 \approx 2530.64$ dollars.



The orange curve (compound interest) bends upward because each year's interest earns interest. The violet line (simple interest) adds the same 80 dollars every year.

7. Continuous compounding

Push the compounding frequency as high as possible, and the formula turns into one that uses e .

P Continuous compounding

If P dollars earn rate r compounded continuously for t years, then

$$A = P e^{rt}.$$

E Example 6: continuous

Same account as before:

$A = 2000 e^{0.04 \cdot 6} = 2000 e^{0.24} \approx 2000 \cdot 1.27125 \approx 2542.50$ dollars. That is only about 3 dollars more than quarterly compounding, which shows how quickly the gains level off.

8. Modeling with exponential functions

M

Finding an exponential model from two points

1. Use the point with $x = 0$ to get a (the y-intercept). If there is no such point, divide the two equations.
2. Divide the outputs and take the right root to find b .
3. Write $f(x) = a \cdot b^x$ and check both points.

E

Example 7: two points

A streaming channel had 150 subscribers at $t = 0$ months and 1,200 at $t = 3$. Then $a = 150$ and $150b^3 = 1200$, so $b^3 = 8$ and $b = 2$. The model is $S(t) = 150 \cdot 2^t$: the channel doubles every month.

Solving for the exponent. To find when 5,000 dollars at 6% compounded yearly doubles, solve $1.06^t = 2$. Taking logarithms gives $t = \frac{\ln 2}{\ln 1.06} \approx 11.9$ years. For continuous growth, $e^{rt} = 2$ gives $t = \frac{\ln 2}{r}$.

Key takeaways

- An exponential function is $f(x) = a \cdot b^x$ with $b > 0$ and $b \neq 1$; equal steps in x multiply y by the same factor.
- $b = 1 + r$ means growth, $b = 1 - r$ means decay; $b > 1$ increases and $0 < b < 1$ decreases.
- The graph has y-intercept $(0, a)$, a horizontal asymptote, domain all reals and range $y > 0$ (for $a > 0$).
- In $a \cdot b^{x-h} + k$, the shifts move the graph h right and k up, and the asymptote becomes $y = k$.
- $e \approx 2.71828$; compound interest is $A = P(1 + r/n)^{nt}$, continuous is $A = Pe^{rt}$.
- To model, find a and b from data; to find a time, use logarithms.



Test yourself: quick challenge for Grade 11

[Change level](#)

Level : **Grade 11**

6 random questions, instant feedback.

[Start the challenge](#)

Speed drill for Grade 11: how many in 60 seconds?

 Keep exploring with Zyro

 Math practice

Exponential Functions: math practice,
Grade 11

 Math tests

Exponential Functions: math test, Grade
11

 Math quizzes

Exponential Functions: math quiz, Grade
11

 Math practice

Logarithmic Functions: math practice,
Grade 11

 Math practice

Sequences and Series: math practice,
Grade 11

 Math tests

Logarithmic Functions: math test, Grade
11

Zyro vous emmène sur nos sites partenaires

Embarquez pour d'autres planètes du savoir et leurs applis.

Décollage