

Exponential and Logarithmic Functions: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Evaluating an exponential function



$$f(0) = 3 \cdot 2^0 = 3 \cdot 1 = 3.$$

$$f(4) = 3 \cdot 2^4 = 3 \cdot 16 = 48.$$

$$f(-1) = 3 \cdot 2^{-1} = 3 \cdot \frac{1}{2} = \frac{3}{2} = 1.5.$$

2 Growth or decay?



(a) $b = 1.08 > 1$: growth of 8% per step.

(b) $b = 0.9 < 1$: decay, $1 - 0.9 = 0.1$, so 10% loss per step.

(c) $b = 1.5 > 1$: growth of 50% per step.

(d) $b = \frac{1}{3} < 1$: decay; the quantity loses two thirds of its value, about 66.7%, per step.

3 From exponential to logarithmic form



Use $b^y = x \iff \log_b x = y$.

(a) $\log_2 128 = 7$.

(b) $\log 0.01 = -2$.

(c) $\log_9 3 = \frac{1}{2}$.

4 Logarithms without a calculator



(a) $3^4 = 81$, so 4.

(b) $5^{-2} = \frac{1}{25}$, so -2 .

(c) $4^{1/2} = 2$, so $\frac{1}{2}$.

(d) $\ln e^5 = 5$ (inverse property).

(e) $10^3 = 1000$, so 3.

5 Condensing logarithms



(a) $\log_6(4 \cdot 9) = \log_6 36 = 2$.

(b) $\log_2 \frac{40}{5} = \log_2 8 = 3$.

(c) $\log_3 \frac{6^2}{4} = \log_3 9 = 2$.

6 Matching bases



(a) $64 = 2^6$, so $x = 6$.

(b) $125 = 5^3$, so $x + 1 = 3$ and $x = 2$.

(c) $0.001 = 10^{-3}$, so $x = -3$.

(d) $8 = 2^3 = \left(\frac{1}{2}\right)^{-3}$, so $x = -3$.

7 Town population



(a) The factor is $1 + 0.03 = 1.03$, so $P(t) = 12000(1.03)^t$.

(b) $P(2) = 12000(1.0609) = 12,730.8 \approx 12,731$.

The town has about **12,731 people** after 2 years.

8 Simple logarithmic equations



(a) $x = 3^4 = 81$.

(b) $x - 1 = 2^3 = 8$, so $x = 9$.

(c) $x = e^0 = 1$.

(d) $x^2 = 49$ and $x > 0$, so $x = 7$.

9 Using the change of base formula



(a) $\log_5 40 = \frac{\ln 40}{\ln 5} \approx \frac{3.68888}{1.60944} \approx 2.2920$.

(b) $\log_2 10 = \frac{\ln 10}{\ln 2} \approx \frac{2.30259}{0.69315} \approx 3.3219$.

10 An exponential equation with a calculator



Take $\ln$ of both sides: $x \ln 7 = \ln 30$.

Exact: $x = \frac{\ln 30}{\ln 7}$. Approximate: $\frac{3.40120}{1.94591} \approx 1.7479$.

Check: $7^{1.7479} \approx 30$. Good.

11 Equation with base e



Divide by 4: $e^{2x} = 9$.

Take **ln**: $2x = \ln 9$, so $x = \frac{\ln 9}{2} = \ln 3 \approx 1.0986$.

12 A logarithmic equation



Exponential form: $3x + 1 = 4^2 = 16$, so $3x = 15$ and $x = 5$.

Check: $3(5) + 1 = 16 > 0$ and $\log_4 16 = 2$. Correct.

13 Monthly compounding



Here $P = 2500$, $r = 0.048$, $n = 12$, $t = 5$, so $\frac{r}{n} = 0.004$.

$A = 2500(1.004)^{60} \approx 2500(1.27064) \approx \$3,176.60$.

14 When does the car lose enough value?



Solve $24000(0.85)^t = 10000$: $0.85^t = \frac{5}{12}$, so $t = \frac{\ln(5/12)}{\ln 0.85} \approx 5.39$.

Check: $V(5) \approx \$10,648.93$ and $V(6) \approx \$9,051.58$.

The car is worth less than \$10,000 during the 6th year; after **6 whole years** it is first below that value.

15 Medicine in the body



(a) $N(10) = 80 \cdot 0.5^{10/6} \approx 80(0.31498) \approx 25.2$ mg.

(b) $10 = 80 \cdot 0.5^{t/6} \Rightarrow 0.5^{t/6} = \frac{1}{8} = 0.5^3 \Rightarrow \frac{t}{6} = 3 \Rightarrow t = 18$.

Ten milligrams remain after **18 hours** (three half-lives), as the graph shows.

16 Expanding a logarithm



Quotient law: $\ln(x^3\sqrt{y}) - \ln e^2$.

Product and power laws: $3\ln x + \frac{1}{2}\ln y - 2$.

Answer: $3\ln x + \frac{1}{2}\ln y - 2$.

17 Equation in quadratic form



Since $9^x = (3^x)^2$, let $u = 3^x > 0$: $u^2 - 4u - 45 = 0 \Rightarrow (u - 9)(u + 5) = 0$.

$u = -5$ is impossible because $3^x > 0$. So $3^x = 9$ and $x = 2$.

Check: $81 - 36 - 45 = 0$.

18 Two logs, one extraneous root



Condense: $\log[x(x - 21)] = 2 \Rightarrow x(x - 21) = 10^2 = 100$.

$x^2 - 21x - 100 = 0 \Rightarrow (x - 25)(x + 4) = 0$.

Domain: $x > 21$, so $x = -4$ is rejected. Answer: $x = 25$.

Check: $\log 25 + \log 4 = \log 100 = 2$.

19 Equating arguments



Condense the left side: $\ln[(x + 2)(x - 2)] = \ln 5 \Rightarrow x^2 - 4 = 5 \Rightarrow x^2 = 9$.

So $x = 3$ or $x = -3$. The domain requires $x > 2$, so $x = -3$ is rejected.

Answer: $x = 3$.

20 Doubling and tripling time

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(a) $Pe^{0.045t} = 2P \Rightarrow t = \frac{\ln 2}{0.045} \approx 15.40$ years.

(b) $Pe^{0.06t} = 3P \Rightarrow t = \frac{\ln 3}{0.06} \approx 18.31$ years.

21 Chemistry: pH

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(a) $\text{pH} = -\log(3.2 \times 10^{-5}) = -(\log 3.2 - 5) \approx -(0.5051 - 5) \approx 4.49$.

(b) $-\log[\text{H}^+] = 7.4 \Rightarrow [\text{H}^+] = 10^{-7.4} \approx 3.98 \times 10^{-8}$ mol/L.

22 Fitting a model to two data points

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(a) $800e^{4k} = 5000 \Rightarrow e^{4k} = 6.25 \Rightarrow k = \frac{\ln 6.25}{4} = \frac{\ln 2.5}{2} \approx 0.4581$.

(b) $N(6) = 800e^{6k} = 800(6.25)^{1.5} = 800(15.625) = 12,500$ cells.

23 True or false?

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(a) **False.** With $a = b = 10$: $\log 20 \approx 1.301$ but $\log 10 + \log 10 = 2$.

(b) **False.** For $x = -3$, $\ln 9 \approx 2.197$ but $\ln(-3)$ does not exist. The correct identity is $\ln(x^2) = 2 \ln |x|$.

(c) **True.** $\log_b x \cdot \log_x b = \frac{\ln x}{\ln b} \cdot \frac{\ln b}{\ln x} = 1$.

24

Annual versus continuous

(a) Solve $800(1.07)^t = 5000 \Rightarrow 1.07^t = 6.25 \Rightarrow t = \frac{\ln 6.25}{\ln 1.07} \approx 27.09$. Check:
 $800(1.07)^{27} \approx \$4,971$ and $800(1.07)^{28} \approx \$5,319$. The balance first passes \$5,000 at the end of year **28**.

(b) $800e^{0.07t} = 5000 \Rightarrow t = \frac{\ln 6.25}{0.07} \approx 26.18$ years.


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