

Eigenvalues and Orthogonality: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Eigenvalues and eigenvectors

/ 4 pts

- (a) Trace **15**, determinant **$56 - 6 = 50$** : $\lambda^2 - 15\lambda + 50$. (1.5 pts)
- (b) $\lambda^2 - 15\lambda + 50 = (\lambda - 5)(\lambda - 10)$, so the eigenvalues are **5** and **10**. (1 pt)
- (c) $\lambda = 5$: $A - 5I = \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$ gives **$(1, -1)$** . $\lambda = 10$: $A - 10I = \begin{pmatrix} -3 & 2 \\ 3 & -2 \end{pmatrix}$ gives **$(2, 3)$** . (1.5 pts)

2 Diagonalization

/ 3 pts

- (a) $\det(M - \lambda I) = (3 - \lambda)[(1 - \lambda)^2 - 4] = (3 - \lambda)(\lambda - 3)(\lambda + 1)$. The eigenvalues are **3** (twice) and **-1**. Check: trace **$5 = 3 + 3 - 1$** . (2 pts)
- (b) Yes. The vectors **$(1, 0, 0)$** and **$(0, 1, 1)$** are independent eigenvectors for **3**, and **$(0, 1, -1)$** is an eigenvector for **-1**. That gives three independent eigenvectors. (1 pt)

3 Dot product and angle

/ 4 pts

(a) $\|\mathbf{u}\| = \sqrt{9 + 16 + 144} = 13$. (1 pt)

(b) $\mathbf{u} \cdot \mathbf{w} = 6 - 4 + 60 = 62$. $\|\mathbf{w}\| = \sqrt{30}$. $\cos \theta = \frac{62}{13\sqrt{30}} \approx 0.8707$, so $\theta \approx 29.5^\circ$. (2 pts)

(c) $3t + 12 + 24 = 0$, so $t = -12$. (1 pt)

4 Orthogonal projection

/ 3 pts

(a) $\mathbf{b} \cdot \mathbf{a} = 13$, $\mathbf{a} \cdot \mathbf{a} = 5$, so $\mathbf{p} = \frac{13}{5}(2, -1) = (5.2, -2.6)$. (1.5 pts)

(b) $\mathbf{b} - \mathbf{p} = (1.8, 3.6)$ and $(1.8)(2) + (3.6)(-1) = 0$. (0.5 pt) The distance is $\sqrt{3.24 + 12.96} = \sqrt{16.2} \approx 4.02$. (1 pt)

5 Gram-Schmidt

/ 3 pts

$\mathbf{u}_1 = (2, 1, 2)$, $\mathbf{u}_1 \cdot \mathbf{u}_1 = 9$ and $\mathbf{v}_2 \cdot \mathbf{u}_1 = 9$. (1 pt)

$\mathbf{u}_2 = (3, 3, 0) - 1 \cdot (2, 1, 2) = (1, 2, -2)$. Check: $2 + 2 - 4 = 0$. (1 pt)

Both vectors have length 3, so the orthonormal pair is $(\frac{2}{3}, \frac{1}{3}, \frac{2}{3})$ and $(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3})$. (1 pt)

6 Least squares

/ 3 pts

$n = 4$, $\sum x = 6$, $\sum y = 24$, $\sum x^2 = 14$, $\sum xy = 0 + 5 + 12 + 30 = 47$. (1 pt)

$b = \frac{4 \cdot 47 - 6 \cdot 24}{4 \cdot 14 - 36} = \frac{44}{20} = 2.2$ and $a = \frac{24 - 13.2}{4} = 2.7$. The line is $y = 2.7 + 2.2x$. (1.5 pts)

For day 4: $2.7 + 8.8 = 11.5$, about 115 bikes. (0.5 pt)



Test yourself: quick challenge for College

[Change level](#)

Level : College

6 random questions, instant feedback.



Start the challenge



Keep exploring with Zyro



Math tests

Eigenvalues and Orthogonality: math test, College



Math lessons

Eigenvalues and Orthogonality: math lesson, College



Math quizzes



Math tests

Eigenvalues and Orthogonality: math quiz, College

Logic, Proofs and Discrete Math: math test, College



Math tests

Limits and Continuity: math test, College



Math lessons

Logic, Proofs and Discrete Math: math lesson, College

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off