

Eigenvalues and Orthogonality: math test, College – download the PDF

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Eigenvalues and eigenvectors

/ 4 pts

Let $A = \begin{pmatrix} 7 & 2 \\ 3 & 8 \end{pmatrix}$.

- Find the characteristic polynomial of A .
- Find the eigenvalues of A .
- Find an eigenvector for each eigenvalue.

2 Diagonalization

/ 3 pts

Let $M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$.

- Find the eigenvalues of M .
- Is M diagonalizable? Justify.

3 Dot product and angle

/ 4 pts

Let $\mathbf{u} = (3, -4, 12)$ and $\mathbf{w} = (2, 1, 5)$.

- Compute $\|\mathbf{u}\|$.
- Compute $\mathbf{u} \cdot \mathbf{w}$ and the angle between the vectors to the nearest tenth of a degree (calculator allowed).
- Find t so that $\mathbf{u}$ is orthogonal to $(t, -3, 2)$.

4 Orthogonal projection

/ 3 pts

Let $\mathbf{a} = (2, -1)$ and $\mathbf{b} = (7, 1)$.

- Compute the projection $\mathbf{p}$ of $\mathbf{b}$ onto the line spanned by $\mathbf{a}$.
- Verify that $\mathbf{b} - \mathbf{p}$ is orthogonal to $\mathbf{a}$, then find the distance from $\mathbf{b}$ to the line (calculator allowed).

5 Gram-Schmidt

/ 3 pts

Apply Gram-Schmidt to $\mathbf{v}_1 = (2, 1, 2)$ and $\mathbf{v}_2 = (3, 3, 0)$, then give an orthonormal pair.

6 Least squares

/ 3 pts

A bike rental shop records the number of bikes rented (in tens) over four days: day 0: 3; day 1: 5; day 2: 6; day 3: 10. Find the least squares line $y = a + bx$ and use it to predict the value for day 4 (calculator allowed).



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