

Logic, Proofs and Discrete Math: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Logic

/ 3 pts

(a) (2 pts)

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$	$p \wedge \neg q$
T	T	T	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	F	F

The last two columns agree in every row, so the statements are equivalent.

(b) (1 pt) “There exists a real number x such that for every integer n , $n \leq x$.”

2 A proof

/ 3 pts

The contrapositive is: if n is even, then n^2 is even. (1 pt)

If $n = 2k$, then $n^2 = 4k^2 = 2(2k^2)$, which is even. (1 pt)

Since the contrapositive is true, the original statement is true. (1 pt)

3 Induction

/ 4 pts

Base (1 pt): for $n = 1$, the left side is 1 and the right side is $\frac{1 \cdot 2 \cdot 3}{6} = 1$.

Hypothesis (1 pt): assume the formula holds for some $n \geq 1$.

Step (1.5 pts): adding $(n + 1)^2$ gives

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{(n+1)[n(2n+1) + 6(n+1)]}{6} = \frac{(n+1)(2n^2 + 7n + 6)}{6}$$

, which is the formula for $n + 1$.

Conclusion (0.5 pt): the formula holds for all $n \geq 1$.

4 Sets and functions

/ 3 pts

(a) (1 pt) Multiples of 2: 6; multiples of 3: 4; multiples of 6: 2. So $6 + 4 - 2 = 8$ integers.

(b) (1 pt) Each of the 3 inputs has 4 possible outputs: $4^3 = 64$ functions.

(c) (1 pt) Injective: outputs must be distinct: $4 \cdot 3 \cdot 2 = 24$ functions.

5 Counting and graphs

/ 4 pts

(a) (1 pt) Order matters: $9 \cdot 8 \cdot 7 = 504$.

(b) (1 pt) $C(9, 4) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{24} = 126$.

(c) (1 pt) $\frac{6 \cdot 5}{2} = 15$ edges.

(d) (1 pt) The degree sum is $3 + 3 + 3 + 3 + 2 + 2 = 16$, so the graph has $16/2 = 8$ edges.

6 A recurrence

/ 3 pts

(a) (1 pt) $a_1 = 5, a_2 = 13, a_3 = 29$.

(b) Base (0.5 pt): $2^2 - 3 = 1 = a_0$. Step (1 pt): if $a_n = 2^{n+2} - 3$, then $a_{n+1} = 2(2^{n+2} - 3) + 3 = 2^{n+3} - 3$. Conclusion (0.5 pt): the formula holds for all $n \geq 0$.



Test yourself: quick challenge for College

Change level

Level : College

6 random questions, instant feedback.



Start the challenge



Keep exploring with Zyro

 Math tests

Logic, Proofs and Discrete Math: math test, College

 Math lessons

Logic, Proofs and Discrete Math: math lesson, College

 Math quizzes

Logic, Proofs and Discrete Math: math quiz, College

 Math tests

Limits and Continuity: math test, College

 Math tests

The Derivative and Its Definition: math test, College

 Math lessons

Limits and Continuity: math lesson, College

Zyro takes you to our partner sites

Blast off to other planets of knowledge and their apps.

Lift off