

Logic, Proofs and Discrete Math: math practice, College – download the PDF

21 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

1 A truth table

★ ★ ★

Build the truth table of $p \wedge \neg q$. For which truth values of p and q is it true?

2 Negating statements

★ ★ ★

Write the negation of each statement. (a) “Every dog in the park is on a leash.” (b) “The pizza is cold and the salad is warm.”

3 Converse and contrapositive

★ ★ ★

Consider: “If an integer is divisible by 6, then it is divisible by 3.” Write its converse and its contrapositive. Which of them is true?

4 Set operations

★ ★ ★

Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 10\}$. Find $A \cap B$, $A \cup B$, $A \setminus B$ and $|A \cup B|$.

5 Outfits



A runner owns 5 shirts, 4 pairs of shorts and 3 pairs of running shoes. How many different outfits can she make with one of each?

6 Committee or officers



A robotics team has 7 members. (a) In how many ways can a 2-person repair crew be chosen? (b) In how many ways can a captain and a co-captain be chosen?

7 First terms of a recurrence



A sequence satisfies $a_1 = 3$ and $a_{n+1} = 2a_n + 1$. Compute $a_2, \dots, a_5$.

8 Sum of two odd integers



Prove directly that the sum of two odd integers is even.

9 Quantifier order



Over the integers, decide whether each statement is true and justify. (a) $\forall x \exists y (x + y = 0)$. (b) $\exists y \forall x (x + y = 0)$. Write the negation of (b).

10 Sum of odd numbers



Prove by induction that $1 + 3 + 5 + \dots + (2n - 1) = n^2$ for all $n \geq 1$.

11 Injective or surjective?



Let $f : \mathbb{Z} \rightarrow \mathbb{Z}$, $f(x) = 2x + 3$, and $g : \mathbb{Z} \rightarrow \mathbb{Z}$, $g(x) = x^2$. Decide whether each is injective and whether each is surjective.

12 Handshake and Euler



A graph has vertices 1 to 5 and edges 1–2, 2–3, 3–4, 4–1, 1–3 and 4–5. List the degrees, check the handshake theorem, and decide whether an Euler path exists.

13 Arranging letters



How many distinct arrangements of all the letters of the word LEVEL are there?

14 Two simple recurrences



(a) A gym has 5 members and gains 4 new members each week, so $a_n = a_{n-1} + 4$, $a_0 = 5$. Find a closed form and a_{10} . (b) A culture of 3 bacteria doubles each hour: $b_n = 2b_{n-1}$, $b_0 = 3$. Find b_8 .

15 Irrationality of the square root of 3



Prove by contradiction that $\sqrt{3}$ is irrational. (You may use: if $3 \mid a^2$ then $3 \mid a$.)

16 An inequality by induction



Prove that $2^n > n^2$ for all integers $n \geq 5$.

17 Three overlapping clubs

★★★

Among 100 first-year students, 60 take computer science (C), 45 take math (M) and 40 take statistics (S). Also 25 take C and M, 20 take C and S, 15 take M and S, and 10 take all three. How many take none of the three?

18 Binary strings without two 1s in a row

★★★

Let s_n be the number of binary strings of length n with no two consecutive 1s. (a) Find s_1 and s_2 . (b) Explain why $s_n = s_{n-1} + s_{n-2}$. (c) Compute s_8 .

19 Round robin and odd degrees

★★★

(a) In a tournament, each of 12 teams plays every other team once. How many games are played? (b) Show that a graph with 7 vertices cannot have every vertex of degree 3.

20 Composition and inverse

★★★

Let $f(x) = 3x - 5$ and $g(x) = x^2 + 1$ on the real numbers. (a) Find $f^{-1}(x)$. (b) Compute $g \circ f$ and $f \circ g$ as formulas. (c) Evaluate both at $x = 2$.

21 A second-order recurrence

★★★

Solve $a_n = 5a_{n-1} - 6a_{n-2}$ with $a_0 = 1$ and $a_1 = 4$. Verify your formula on a_2 and a_3 .



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