

Dilations and Similarity: math lesson, Grade 8 – download the PDF

Photo apps, map zoom buttons, and movie projectors all use the same trick: they make a picture bigger or smaller **without changing its shape**. In geometry, that trick is called a **dilation**. In this chapter you will learn how dilations work, how to compute them on the coordinate plane, and how they lead to the big idea of **similar figures**. You will also review the angle facts about triangles and parallel lines that let you prove two triangles are similar.

1. What Is a Dilation?

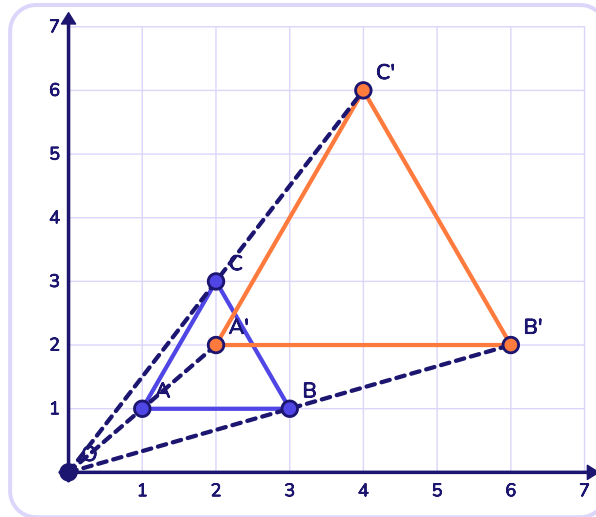
D Dilation

A **dilation** is a transformation defined by a fixed point O , called the **center**, and a positive number k , called the **scale factor**. Each point P is sent to the point P' on the ray OP such that

$$OP' = k \cdot OP.$$

The center O does not move.

Think of a flashlight shining on a wall: your hand is the original figure, the shadow is the image, and the flashlight is the center. The farther the wall is from the lamp, the bigger the shadow.



In the figure, the center is the origin O and the scale factor is **2**. Each dashed ray starts at O , goes through a vertex of triangle ABC , and continues to the matching vertex of triangle $A'B'C'$. Notice that $OA' = 2 \cdot OA$, $OB' = 2 \cdot OB$, and $OC' = 2 \cdot OC$.

2. The Scale Factor

The scale factor k tells you what kind of dilation you have.

Scale factor	Effect	Name
$k > 1$	The image is larger	Enlargement
$0 < k < 1$	The image is smaller	Reduction
$k = 1$	The image is identical	No change

P**What a dilation does**

- Every length is multiplied by k .
- Every angle keeps exactly the same measure.
- A segment is sent to a parallel segment (or to itself if its line passes through the center).

E**Example 1: Using the scale factor**

(a) A point P is **6** cm from the center O . After a dilation with $k = 2.5$,
 $OP' = 2.5 \times 6 = 15$ cm.

(b) A side of a figure measures **12** inches. After a dilation with $k = \frac{3}{4}$, the new side is
 $\frac{3}{4} \times 12 = 9$ inches, so the figure shrinks.

Common mistake

A dilation **multiplies** lengths by k ; it does not add k . Doubling a side of **5** cm gives **10** cm because $5 \times 2 = 10$, not because you add **2**. And a dilation never changes angle measures.

3. Dilations on the Coordinate Plane

When the center of the dilation is the origin $(0, 0)$, the rule is very simple.

P**Dilation centered at the origin**

The dilation with center $(0, 0)$ and scale factor k sends

$$(x, y) \longrightarrow (kx, ky).$$

If the center is a point (a, b) , use $(x, y) \rightarrow (a + k(x - a), b + k(y - b))$.

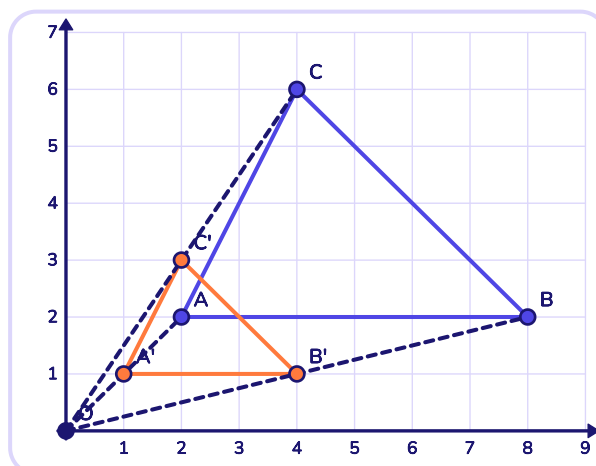
M**Method**

1. Write the coordinates of each vertex.
2. Multiply both coordinates of each vertex by k .
3. Plot the new vertices and connect them in the same order.
4. Check that each new side is k times the old one.

E**Example 2: An enlargement**

Dilate $A(1, 1)$, $B(3, 1)$, $C(2, 3)$ by $k = 2$ from the origin: $A'(2, 2)$, $B'(6, 2)$, $C'(4, 6)$.

Side $AB = 2$ and $A'B' = 4$, which is twice as long, as expected (see the first figure).



E**Example 3: A reduction**

Dilate $A(2, 2)$, $B(8, 2)$, $C(4, 6)$ by $k = \frac{1}{2}$: $A'(1, 1)$, $B'(4, 1)$, $C'(2, 3)$. The figure shows the smaller triangle sitting inside the larger one, with the same shape.

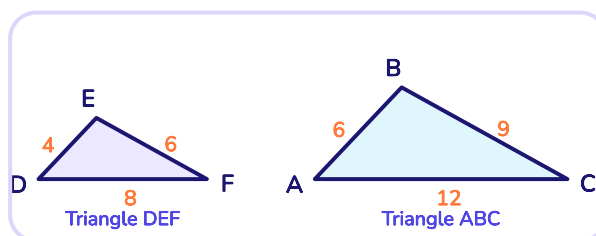
!**Zyro's tip**

On my home planet we say: "The origin is the lamp, and the scale factor is the zoom." Multiply both coordinates by the zoom and the whole picture follows!

4. Similar Figures**D****Similar figures**

Two figures are **similar** if one can be obtained from the other by a sequence of rotations, reflections, translations, and dilations. We write $\triangle ABC \sim \triangle DEF$.

If two figures are similar, then their **corresponding angles are equal** and their **corresponding sides are proportional**: all the ratios of matching sides are equal to the same number, the scale factor.



E**Example 4: Checking similarity**

Triangle DEF has sides **4, 6, 8** and triangle ABC has sides **6, 9, 12**. Compare the matching sides:

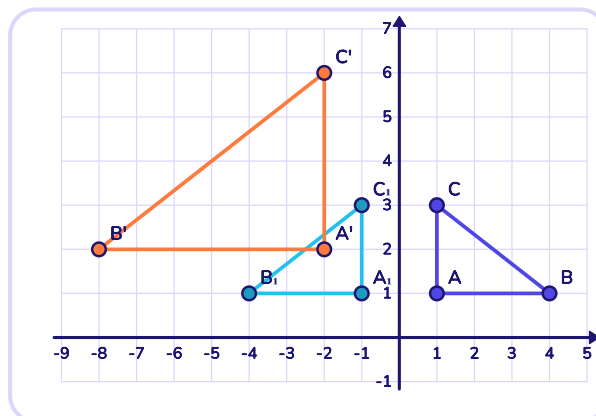
$$\frac{6}{4} = 1.5, \quad \frac{9}{6} = 1.5, \quad \frac{12}{8} = 1.5.$$

All ratios are equal, so the triangles are similar with scale factor **1.5**.

A rectangle 4×10 and a rectangle 6×14 are **not** similar, because $\frac{6}{4} = 1.5$ but $\frac{14}{10} = 1.4$.

5. Sequences of Transformations and Similarity

Rotations, reflections, and translations keep every length and angle: the image is **congruent** to the original. A dilation with $k \neq 1$ changes the size. When a sequence of transformations includes at least one dilation, the image is **similar** but usually not congruent to the original.



E**Example 5: Describing a sequence**

Triangle ABC with $A(1, 1)$, $B(4, 1)$, $C(1, 3)$ is first reflected across the y -axis, which gives $A_1(-1, 1)$, $B_1(-4, 1)$, $C_1(-1, 3)$. Then it is dilated by $k = 2$ from the origin, which gives $A'(-2, 2)$, $B'(-8, 2)$, $C'(-2, 6)$. Since the sequence has a reflection and a dilation, $\triangle A'B'C' \sim \triangle ABC$ with scale factor **2**: for instance $AB = 3$ and $A'B' = 6$.

6. Angles in Triangles and Parallel Lines**P****Triangle angle sum**

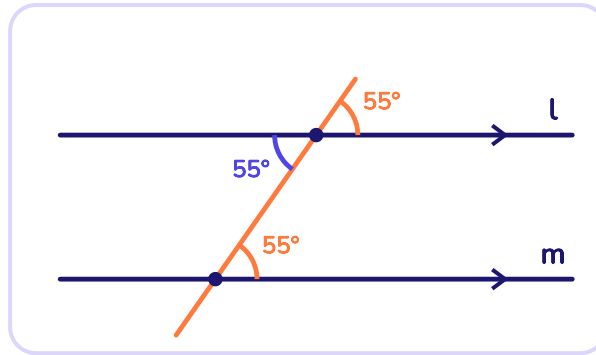
The three interior angles of any triangle add up to **180°** .

Why? Draw a line through one vertex parallel to the opposite side. The two angles formed next to the triangle are equal to two of the triangle's angles (alternate interior angles), and together with the third angle they form a straight line, which measures **180°** .

P**Parallel lines and a transversal**

When two parallel lines are cut by a transversal:

- **corresponding angles** are equal;
- **alternate interior angles** are equal;
- **same-side interior angles** add up to **180°** ;
- vertical angles are always equal.



E Example 6: Using the angle sum

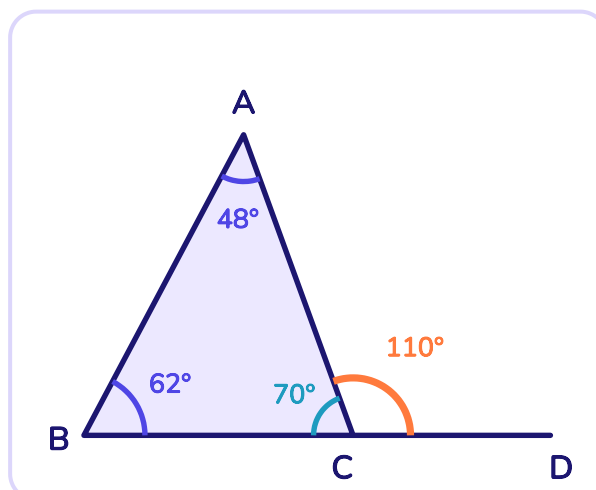
A triangle has angles 47° , $2x^\circ$, and $(x + 13)^\circ$. Then $47 + 2x + x + 13 = 180$, so $3x = 120$ and $x = 40$. The angles are 47° , 80° , and 53° , and indeed $47 + 80 + 53 = 180$.

7. The Exterior Angle Theorem

If you extend one side of a triangle, the angle formed outside the triangle is an **exterior angle**.

P Exterior Angle Theorem

An exterior angle of a triangle is equal to the **sum of the two remote interior angles** (the two interior angles that are not next to it).



Here is the proof. The exterior angle at C and the interior angle at C form a straight line, so the exterior angle is $180^\circ - \angle C$. But the angle sum gives $\angle A + \angle B = 180^\circ - \angle C$. Both quantities are equal! In the figure, $48^\circ + 62^\circ = 110^\circ$, and also $180^\circ - 70^\circ = 110^\circ$.

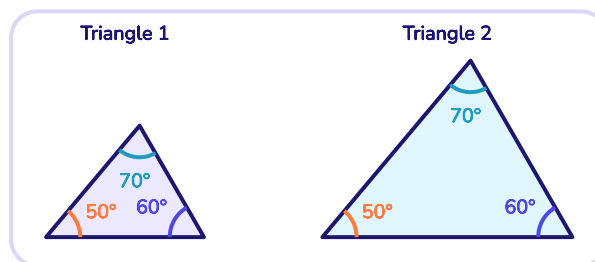
8. The Angle-Angle (AA) Criterion

P

AA criterion

If two angles of one triangle are equal to two angles of another triangle, then the triangles are **similar**.

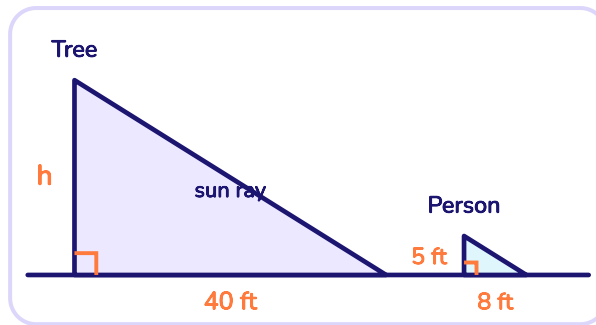
The reason is the angle sum: once two angles match, the third angles must match too (each is 180° minus the other two). All three pairs of angles are then equal, so one triangle is a scaled copy of the other. In the figure, both triangles have angles of 50° , 60° , and 70° .



M

Method: Indirect measurement

1. Find two right triangles (or other triangles) that share two equal angles, for example because the sun's rays are parallel.
2. State that they are similar by AA.
3. Write a proportion with matching sides.
4. Solve for the unknown length.



E Example 7: How tall is the tree?

A 5-foot-tall person (about 1.5 m) casts an 8-foot shadow. At the same time a tree casts a 40-foot shadow. Both form right triangles and the sun's angle is the same, so the triangles are similar by AA. Then

$$\frac{h}{40} = \frac{5}{8} \Rightarrow h = \frac{5 \times 40}{8} = 25.$$

The tree is 25 feet tall, about 7.6 meters.

Key takeaways

- A dilation with center O and scale factor k sends P to P' on ray OP with $OP' = k \cdot OP$.
- $k > 1$ enlarges, $0 < k < 1$ reduces; lengths are multiplied by k and angles do not change.
- On the coordinate plane with center at the origin: $(x, y) \rightarrow (kx, ky)$.
- Similar figures have equal corresponding angles and proportional corresponding sides.
- A sequence of rotations, reflections, translations, and dilations produces a similar figure.
- A triangle's angles add up to 180° ; an exterior angle equals the sum of the two remote interior angles.
- Two triangles with two pairs of equal angles are similar (AA).



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