

# Differentiation Rules: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

## 1 Product and quotient rules

/ 4 pts

(a)  $f'(x) = 3(x^2 + 4) + (3x - 1)(2x) = 3x^2 + 12 + 6x^2 - 2x = 9x^2 - 2x + 12$ . (2 pts)

(b)  $g'(x) = \frac{2(x - 3) - (2x + 1)(1)}{(x - 3)^2} = \frac{-7}{(x - 3)^2}$ . (2 pts)

## 2 Chain rule

/ 3 pts

(a)  $y' = 6(x^3 + 2)^5 \cdot 3x^2 = 18x^2(x^3 + 2)^5$ . (1.5 pts)

(b) Product and chain rules:

$y' = -3e^{-3x} \cos(2x) + e^{-3x}(-2 \sin(2x)) = -e^{-3x}(3 \cos(2x) + 2 \sin(2x))$ . (1.5 pts)

### 3 A logarithmic quotient

/ 3 pts

(a)  $y' = \frac{\frac{1}{x} \cdot x^2 - \ln x \cdot 2x}{x^4} = \frac{x - 2x \ln x}{x^4} = \frac{1 - 2 \ln x}{x^3}$ . (2 pts)

(b)  $y' = 0$  when  $1 - 2 \ln x = 0$ , so  $\ln x = \frac{1}{2}$  and  $x = \sqrt{e}$ . (1 pt)

### 4 Implicit differentiation

/ 3 pts

Check:  $27 + 27 = 54 = 6 \cdot 9$ . Differentiating:  $3x^2 + 3y^2 y' = 6y + 6x y'$ . (1 pt)

So  $y'(3y^2 - 6x) = 6y - 3x^2$ , giving  $y' = \frac{2y - x^2}{y^2 - 2x}$ . (1 pt)

At  $(3, 3)$ :  $y' = \frac{6 - 9}{9 - 6} = -1$ , so the tangent is  $y - 3 = -(x - 3)$ , i.e.  $y = -x + 6$ . (1 pt)

### 5 Logarithmic differentiation

/ 4 pts

(a)  $\ln y = 3x \ln x$ , so  $\frac{y'}{y} = 3 \ln x + 3$  and  $y' = 3x^{3x}(\ln x + 1)$ . (2 pts) At  $x = 1$ :

$y'(1) = 3 \cdot 1 \cdot (0 + 1) = 3$ . (1 pt)

(b)  $\frac{y'}{y} = \frac{4}{x+1} + \frac{1}{2(x-2)} - \frac{6}{3x-1}$ ; at  $x = 3$ :  $1 + \frac{1}{2} - \frac{3}{4} = \frac{3}{4}$ . (1 pt)

### 6 Higher derivatives and concavity

/ 3 pts

(a)  $f'(x) = e^{-x} - x e^{-x} = (1 - x)e^{-x}$ . (1 pt)

(b)  $f''(x) = -e^{-x} - (1 - x)e^{-x} = (x - 2)e^{-x}$ . (1 pt)

(c) Since  $e^{-x} > 0$ ,  $f'' = 0$  only at  $x = 2$ ;  $f'' > 0$  for  $x > 2$  (bends upward) and  $f'' < 0$  for  $x < 2$ . (1 pt)



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