

Differentiation Rules: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Product rule on polynomials



Take $u = x^2 + 1$, $v = x^3 - 2x$; then $u' = 2x$ and $v' = 3x^2 - 2$.

$$f'(x) = 2x(x^3 - 2x) + (x^2 + 1)(3x^2 - 2) = 2x^4 - 4x^2 + 3x^4 - 2x^2 + 3x^2 - 2 = 5x^4 -$$

Check: $f(x) = x^5 - 2x^3 + x^3 - 2x = x^5 - x^3 - 2x$, whose derivative is $5x^4 - 3x^2 - 2$.

Both methods agree.

2 A first quotient



$$g'(x) = \frac{1 \cdot (x + 3) - x \cdot 1}{(x + 3)^2} = \frac{3}{(x + 3)^2}.$$

$$\text{At } x = 1: g'(1) = \frac{3}{16}.$$

3 Power of a linear function



Outer: u^5 ; inner: $u = 4x - 7$ with $u' = 4$.

$$h'(x) = 5(4x - 7)^4 \cdot 4 = 20(4x - 7)^4. \text{ At } x = 2, 4x - 7 = 1, \text{ so } h'(2) = 20.$$

4 Trigonometric sum



$$y' = 3 \cos x + 2 \sin x + \sec^2 x.$$

$$\text{At } x = 0: 3(1) + 2(0) + 1 = 4.$$

5 Exponentials and logarithms



$$f'(x) = 5e^x - \frac{4}{x} + 2^x \ln 2.$$

$$f'(1) = 5e - 4 + 2 \ln 2 \approx 13.59 - 4 + 1.39 = 10.98.$$

6 Successive derivatives



$$f'(x) = 4x^3 - 9x^2 + 2; f''(x) = 12x^2 - 18x; f^{(3)}(x) = 24x - 18; f^{(4)}(x) = 24.$$

$$f''(2) = 12(4) - 18(2) = 48 - 36 = 12.$$

7 Velocity and acceleration of a cart



$$\text{Velocity: } v(t) = s'(t) = 3t^2 - 12t + 9. \text{ Acceleration: } a(t) = s''(t) = 6t - 12.$$

$$\text{At } t = 4: v = 48 - 48 + 9 = 9 \text{ ft/s and } a = 24 - 12 = 12 \text{ ft/s}^2.$$

$$\text{At rest when } 3(t^2 - 4t + 3) = 3(t - 1)(t - 3) = 0, \text{ that is at } t = 1 \text{ s and } t = 3 \text{ s.}$$

8 Product with a sine



$$f'(x) = 2x \sin x + x^2 \cos x.$$

$$\text{At } x = \pi: 2\pi(0) + \pi^2(-1) = -\pi^2 \approx -9.87.$$

9 Where is the tangent horizontal?



$$y' = \frac{e^x(x^2 + 1) - e^x(2x)}{(x^2 + 1)^2} = \frac{e^x(x^2 - 2x + 1)}{(x^2 + 1)^2} = \frac{e^x(x - 1)^2}{(x^2 + 1)^2}.$$

The denominator is never 0 and $e^x > 0$, so $y' = 0$ only when $x = 1$. The point is $\left(1, \frac{e}{2}\right)$.

Since $y' \geq 0$ everywhere, the function never decreases.

10 Cosine of a quadratic



Outer: $\cos u$; inner: $u = 3x^2 + 1$, $u' = 6x$. So

$$y' = -\sin(3x^2 + 1) \cdot 6x = -6x \sin(3x^2 + 1).$$

At $x = 0$: $y' = 0$. At $x = 1$: $y' = -6 \sin 4 \approx -6(-0.7568) \approx 4.54$.

11 Logarithm of a quadratic



$x^2 + 4x + 5 = (x + 2)^2 + 1 \geq 1 > 0$, so the logarithm is defined for all real x .

$$y' = \frac{2x + 4}{x^2 + 4x + 5}. \text{ This is 0 only when } 2x + 4 = 0, \text{ that is } x = -2.$$

12 Tangent line to an exponential product



$$f'(x) = e^{-2x} + x(-2)e^{-2x} = (1 - 2x)e^{-2x}.$$

(a) $f(0) = 0$ and $f'(0) = 1$, so the tangent is $y = x$.

(b) $f'(x) = 0$ when $1 - 2x = 0$, so $x = \frac{1}{2}$. The point is $\left(\frac{1}{2}, \frac{1}{2e}\right) \approx (0.5, 0.18)$.

13 Implicit slope of a tilted curve



Check: $4 + 2 + 1 = 7$. Differentiate: $2x + y + x y' + 2y y' = 0$, so

$$y'(x + 2y) = -(2x + y) \text{ and } y' = -\frac{2x + y}{x + 2y}.$$

At $(2, 1)$: $y' = -\frac{5}{4}$. Tangent: $y - 1 = -\frac{5}{4}(x - 2)$, i.e. $y = -\frac{5}{4}x + \frac{7}{2}$.

14 Derivatives of $x \ln x$



$$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1; f''(x) = \frac{1}{x}; f^{(3)}(x) = -\frac{1}{x^2}.$$

$$f''(4) = \frac{1}{4} \text{ and } f^{(3)}(2) = -\frac{1}{4}.$$

15 Find the mistakes



(i) The student differentiated only x^2 and ignored the product rule. Correct:

$$2x e^x + x^2 e^x = x(x + 2)e^x.$$

(ii) The inner function $4x$ was ignored (chain rule). Correct: $\cos(4x) \cdot 4 = 4 \cos(4x)$.

16 A variable exponent



$$\ln y = \sin x \cdot \ln x. \text{ Differentiating: } \frac{y'}{y} = \cos x \ln x + \frac{\sin x}{x}.$$

$$\text{So } y' = x^{\sin x} \left(\cos x \ln x + \frac{\sin x}{x} \right).$$

$$\text{At } x = \frac{\pi}{2}: y = \frac{\pi}{2}, \cos = 0, \sin = 1, \text{ so } y' = \frac{\pi}{2} \cdot \frac{2}{\pi} = 1.$$

17 A long product and quotient

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$$\ln y = 3 \ln(x^2 + 1) + 4 \ln(2x - 1) - 2 \ln(x + 5), \text{ so } \frac{y'}{y} = \frac{6x}{x^2 + 1} + \frac{8}{2x - 1} - \frac{2}{x + 5}.$$

$$\text{At } x = 1: y = \frac{8 \cdot 1}{36} = \frac{2}{9} \text{ and } \frac{y'}{y} = 3 + 8 - \frac{1}{3} = \frac{32}{3}.$$

$$\text{Therefore } y'(1) = \frac{2}{9} \cdot \frac{32}{3} = \frac{64}{27}.$$

18 Three layers

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$$\text{Layers: } \sqrt{u}, u = \sin w, w = x^2. \text{ So } y' = \frac{1}{2\sqrt{\sin(x^2)}} \cdot \cos(x^2) \cdot 2x = \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}}.$$

$$\text{At } x = \sqrt{\pi/6}: x^2 = \frac{\pi}{6}, \sin = \frac{1}{2}, \cos = \frac{\sqrt{3}}{2}. \text{ Then}$$

$$y' = \sqrt{\frac{\pi}{6}} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{2} = \sqrt{\frac{\pi}{6}} \cdot \frac{\sqrt{6}}{2} = \frac{\sqrt{\pi}}{2} \approx 0.89.$$

19 Second derivative of a circle

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$$y'' = -\frac{1 \cdot y - x y'}{y^2} = -\frac{y + x^2/y}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{25}{y^3}.$$

At $(3, 4)$: $y'' = -\frac{25}{64}$. Since $y'' < 0$, the circle bends downward there, as expected on the upper half.

20 Inflating balloon

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$$\text{Both } V \text{ and } r \text{ depend on time } t. \text{ Chain rule: } \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

With $r = 6$ and $\frac{dr}{dt} = 0.5$: $\frac{dV}{dt} = 4\pi(36)(0.5) = 72\pi \approx 226.2$ cubic inches per second (about 3,707 cm³/s).

21 Proving the tangent rule



$$(\tan x)' = \frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}.$$

Since $\cos^2 x + \sin^2 x = 1$, this equals $\frac{1}{\cos^2 x} = \sec^2 x$.

22 Product of three functions



For three factors, $(uvw)' = u'vw + uv'w + uvw'$. Here

$$y' = e^x \sin x + x e^x \sin x + x e^x \cos x = e^x (\sin x + x \sin x + x \cos x).$$

$$\text{At } x = \frac{\pi}{2}: y' = e^{\pi/2} \left(1 + \frac{\pi}{2} + 0\right) = e^{\pi/2} \left(1 + \frac{\pi}{2}\right) \approx 12.35.$$



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