

The Derivative and Its Definition: math test solutions, College – download the PDF

Test solutions with the detailed point scale. Add up your points and spot what to review.

Suggested time: 45 minutes. Out of 20 points. Calculator allowed only when the problem says so.

1 Definition of the derivative

/ 4 pts

- (a) $f(1) = 1$; $f(1 + h) = 2(1 + 2h + h^2) - 1 - h = 1 + 3h + 2h^2$. The quotient is $\frac{3h + 2h^2}{h} = 3 + 2h$. (2 pts)
- (b) As $h \rightarrow 0$, $3 + 2h \rightarrow 3$, so $f'(1) = 3$. (1 pt)
- (c) $f'(x) = 4x - 1$, and $f'(1) = 3$. (1 pt)

2 Differentiating

/ 4 pts

- (a) $g'(x) = 20x^3 - 6x^2 + 1$. (1 pt)
- (b) $h(x) = 3x^{1/2} + 4x^{-2}$, so $h'(x) = \frac{3}{2\sqrt{x}} - \frac{8}{x^3}$. (2 pts)
- (c) Expand: $k(x) = x^2 + 4x$, so $k'(x) = 2x + 4$. (1 pt)

3 Tangent lines of a cubic

/ 4 pts

(a) $f(2) = 1$, $f'(x) = 3x^2 - 4x$, $f'(2) = 4$. Tangent: $y = 1 + 4(x - 2) = 4x - 7$. (2 pts)

(b) $3x^2 - 4x = x(3x - 4) = 0$ gives $x = 0$ or $x = \frac{4}{3}$. $f(0) = 1$ and $f\left(\frac{4}{3}\right) = \frac{64}{27} - \frac{96}{27} + \frac{27}{27} = -\frac{5}{27}$. Points: $(0, 1)$ and $\left(\frac{4}{3}, -\frac{5}{27}\right)$. (2 pts)

4 Filling a tank

/ 3 pts

(a) $V(2) = 10$, $V(6) = 42$; average rate = $\frac{32}{4} = 8$ gal/min. (1 pt)

(b) $V'(t) = t + 4$; $V'(2) = 6$ gal/min and $V'(6) = 10$ gal/min. (1 pt)

(c) The tank fills faster and faster; the average 8 gal/min lies between the instantaneous rates 6 and 10. (1 pt)

5 A differentiable piecewise function

/ 3 pts

Continuity at 2: $f(2) = 5$, so $2m + n = 5$. (1 pt)

Equal slopes: left derivative $2x = 4$ at 2, so $m = 4$. (1 pt)

Then $n = 5 - 8 = -3$. (1 pt)

6 Continuity and differentiability

/ 2 pts

(a) True: if $f'(a)$ exists, $f(a + h) - f(a) = \frac{f(a + h) - f(a)}{h} \cdot h \rightarrow f'(a) \cdot 0 = 0$. (1 pt)

(b) False: $|x|$ is continuous at 0 but its quotients tend to 1 and -1 . (1 pt)



Test yourself: quick challenge for College

[Change level](#)

Level : College

6 random questions, instant feedback.



Start the challenge



Keep exploring with Zyro



Math tests

The Derivative and Its Definition: math test, College



Math lessons

The Derivative and Its Definition: math lesson, College



Math quizzes



Math tests

The Derivative and Its Definition: math quiz, College

Differentiation Rules: math test, College

 **Math tests**

Applications of Derivatives: math test, College

 **Math lessons**

Differentiation Rules: math lesson, College

Zyro vous emmène sur nos sites partenaires

Embarquez pour d'autres planètes du savoir et leurs applis.

Décollage