

The Derivative and Its Definition: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Average rate of change



$f(1) = 2$ and $f(4) = 17$. The average rate is $\frac{f(4) - f(1)}{4 - 1} = \frac{15}{3} = 5$.

Answer: 5.

2 A linear difference quotient



$f(x + h) - f(x) = 5(x + h) - 2 - (5x - 2) = 5h$. So the quotient is $\frac{5h}{h} = 5$ for every $h \neq 0$, and $f'(x) = 5$. A line has the same slope everywhere.

3 Power rule practice



(a) $7x^6$. (b) $10x^9$. (c) $6 \cdot 4x^3 = 24x^3$, using the constant multiple rule.

4 A quadratic polynomial



Term by term: $f'(x) = 3 \cdot 2x - 8 \cdot 1 + 0 = 6x - 8$.

5 Skateboard distance



$s(1) = 4$ and $s(3) = 36$. Average velocity $= \frac{36 - 4}{3 - 1} = 16$ ft/s.

In meters: $16 \times 0.3048 \approx 4.88$ m/s.

Answer: 16 ft/s (about 4.88 m/s).

6 True or false?



(a) False: the derivative of a constant is **0**. (b) True: constant multiple rule, $5 \cdot 1 = 5$. (c) False: the exponent drops by one, $4x^3$. (d) False: $|x|$ is continuous at **0** but not differentiable there.

7 Slope of a tangent



$f'(x) = 2x$, so the slope is $f'(3) = 6$. The point is $(3, 9)$. The tangent is $y = 9 + 6(x - 3) = 6x - 9$.

8 Derivative at a point from the definition



$f(2) = -4$. $f(2 + h) = 4 + 4h + h^2 - 8 - 4h = h^2 - 4$. So $f(2 + h) - f(2) = h^2$ and the quotient is h . Its limit as $h \rightarrow 0$ is **0**.

Answer: $f'(2) = 0$ (horizontal tangent at the vertex).

9 Tangent line to a cubic



$f(1) = 1$. $f'(x) = 6x^2 - 1$, so $f'(1) = 5$. Tangent: $y = 1 + 5(x - 1) = 5x - 4$.

10 Temperature during the day



(a) $T'(t) = -t + 6$.

(b) $T'(2) = 4$: at $t = 2$ the temperature is rising at 4 °F per hour. $T'(8) = -2$: at $t = 8$ it is falling at 2 °F per hour.

(c) $T'(t) = 0$ at $t = 6$, where the rate changes from positive to negative.

$$T(6) = -18 + 36 + 50 = 68 \text{ °F, and } \frac{5}{9}(68 - 32) = 20 \text{ °C.}$$

11 Horizontal tangents



A horizontal tangent means $f'(x) = 0$. $f'(x) = 3x^2 - 12 = 3(x - 2)(x + 2)$, so $x = 2$ or $x = -2$.

$$f(2) = 8 - 24 = -16 \text{ and } f(-2) = -8 + 24 = 16.$$

Answer: the points $(2, -16)$ and $(-2, 16)$, as in the figure.

12 Powers and roots



Rewrite $f(x) = x^{-2} + 3x^{1/2}$. Then $f'(x) = -2x^{-3} + \frac{3}{2}x^{-1/2} = -\frac{2}{x^3} + \frac{3}{2\sqrt{x}}$.

$$\text{At } x = 4: -\frac{2}{64} + \frac{3}{4} = -\frac{1}{32} + \frac{24}{32} = \frac{23}{32}.$$

13 A corner



$f(2) = 0$, so the quotient is $\frac{|h|}{h}$. For $h > 0$ it equals 1; for $h < 0$ it equals -1 . The one-sided limits are 1 and -1 ; they differ, so the limit does not exist.

Conclusion: $f'(2)$ does not exist (the graph has a corner at $(2, 0)$).

14 Where is the slope 4?

$f'(x) = 2x - 6$. Solve $2x - 6 = 4$: $x = 5$. Then $f(5) = 25 - 30 + 1 = -4$.

Answer: the point $(5, -4)$.

15 Definition applied to a reciprocal

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \frac{x - (x+h)}{hx(x+h)} = \frac{-h}{hx(x+h)} = \frac{-1}{x(x+h)}. \text{ As } h \rightarrow 0 \text{ this tends to } -\frac{1}{x^2}.$$

So $f'(x) = -\frac{1}{x^2}$, consistent with the power rule for x^{-1} . $f'(2) = -\frac{1}{4}$, $f(2) = \frac{1}{2}$.

Tangent: $y = \frac{1}{2} - \frac{1}{4}(x - 2) = -\frac{1}{4}x + 1$.

16 Making a piecewise function differentiable

Continuity is needed: $f(1) = 1$, so $a + b = 1$.

Equal slopes: the left derivative is $(x^2)' = 2x = 2$ at 1; the right derivative is a . So $a = 2$.

Then $b = 1 - 2 = -1$.

Answer: $a = 2$, $b = -1$, i.e. $f(x) = 2x - 1$ for $x > 1$.

17 Marginal cost

(a) $C'(q) = 0.04q + 3$.

(b) $C'(100) = 4 + 3 = 7$ dollars per unit.

$C(100) = 200 + 300 + 500 = 1000$ and

$C(101) = 0.02 \cdot 10201 + 303 + 500 = 204.02 + 303 + 500 = 1007.02$. The extra cost of the 101st unit is \$7.02, very close to $C'(100) = \$7$.

18 Tangent lines through an outside point

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The tangent at $x = a$ is $y = a^2 + 2a(x - a) = 2ax - a^2$. It passes through $(0, -4)$ when $-a^2 = -4$, so $a = 2$ or $a = -2$.

Answer: $y = 4x - 4$ (tangent at $(2, 4)$) and $y = -4x - 4$ (tangent at $(-2, 4)$).

19 Parallel tangents

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Parallel means slope 12. $3x^2 = 12$ gives $x = \pm 2$.

At $x = 2$: point $(2, 8)$, tangent $y = 8 + 12(x - 2) = 12x - 16$. At $x = -2$: point $(-2, -8)$, tangent $y = -8 + 12(x + 2) = 12x + 16$.

20 A ball thrown upward

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(a) $v(t) = -32t + 96$.

(b) $v(1) = 64$ ft/s ≈ 19.5 m/s.

(c) At the top the velocity is 0: $-32t + 96 = 0$, $t = 3$ s. $h(3) = -144 + 288 + 10 = 154$ ft ≈ 46.9 m.

(d) $v(5) = -160 + 96 = -64$ ft/s. Negative: the ball is on its way down at 64 ft/s.

21 A vertical tangent

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f is continuous at 0 because cube roots are continuous. The quotient is

$\frac{h^{1/3} - 0}{h} = h^{-2/3} = \frac{1}{h^{2/3}}$, which grows without bound as $h \rightarrow 0$ (for $h = 0.001$ it equals 100; for $h = 0.000001$ it equals 10,000). The limit is infinite, so $f'(0)$ does not exist.

Conclusion: the graph has a **vertical tangent** (the y -axis) at the origin.



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