

Conic Sections and Parametric Equations: practice solutions, Grade 12 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Center and radius



Compare with $(x - h)^2 + (y - k)^2 = r^2$: $h = 4$, $k = -7$ (because $y + 7 = y - (-7)$) and $r^2 = 49$.

The center is $(4, -7)$ and the radius is 7.

2 Writing a circle equation



$h = -2$, $k = 5$, $r = 3$, so $(x - (-2))^2 + (y - 5)^2 = 3^2$.

$(x + 2)^2 + (y - 5)^2 = 9$.

3 A simple parabola



The form is $x^2 = 4py$, so $4p = 8$ and $p = 2$.

Vertex $(0, 0)$, focus $(0, 2)$, directrix $y = -2$.

4 Vertices of an ellipse



$a^2 = 49$, so $a = 7$; $b^2 = 25$, so $b = 5$. The long axis is horizontal.

Vertices $(\pm 7, 0)$, co-vertices $(0, \pm 5)$. $c^2 = 49 - 25 = 24$, so $c = \sqrt{24} = 2\sqrt{6} \approx 4.90$.

5 Hyperbola features



$a = 3$, $b = 4$, $c^2 = 9 + 16 = 25$, so $c = 5$.

Vertices $(\pm 3, 0)$, foci $(\pm 5, 0)$, asymptotes $y = \pm \frac{4}{3}x$.

6 Name that conic



a. Sum of squares with equal coefficients: a **circle** of radius **6**.

b. Dividing by **36**: $\frac{x^2}{9} + \frac{y^2}{4} = 1$, an **ellipse**.

c. Difference of squares equal to **1**: a **hyperbola** opening up and down.

d. Only one variable is squared: a **parabola** ($x^2 = \frac{1}{2}y$, so $p = \frac{1}{8}$).

7 A table of points



$t = 0$: $(2, -1)$; $t = 1$: $(3, 2)$; $t = 2$: $(4, 5)$; $t = 3$: $(5, 8)$.

From $x = t + 2$, $t = x - 2$. Then $y = 3(x - 2) - 1 = 3x - 7$. The curve is the line $y = 3x - 7$.

8 Completing the square



Group: $(x^2 + 8x) + (y^2 - 6y) = -9$. Add **16** and **9** to both sides:

$$(x + 4)^2 + (y - 3)^2 = -9 + 16 + 9 = 16.$$

Center $(-4, 3)$, radius **4**.

9 Circle through a point



The radius is the distance: $r = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{25} = 5$.

Standard form: $(x-1)^2 + (y-2)^2 = 25$. Expanding: $x^2 - 2x + 1 + y^2 - 4y + 4 = 25$, so the general form is $x^2 + y^2 - 2x - 4y - 20 = 0$.

10 From focus and directrix



The vertex is halfway between them: $(0, 0)$, and $p = 5$. So $x^2 = 4 \cdot 5 y$, that is $x^2 = 20y$.

Check: $10^2 = 100 = 20 \cdot 5$. The distance to the focus is **10** and the distance to the directrix is $5 - (-5) = 10$.

11 Satellite dish



The equation is $x^2 = 4py$. The rim point is $(5, 2)$ (half the width, full depth):

$$25 = 4p \cdot 2 = 8p.$$

So $p = \frac{25}{8} = 3.125$. The receiver is 3.125 ft (about 0.95 m) above the vertex.

12 Ellipse from foci and vertices



$a = 5, c = 3$, so $b^2 = a^2 - c^2 = 25 - 9 = 16$.

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \text{ and } e = \frac{c}{a} = \frac{3}{5} = 0.6.$$

13 Hyperbola from foci and vertices



$a = 6, c = 10$, so $b^2 = c^2 - a^2 = 100 - 36 = 64$ and $b = 8$.

Equation: $\frac{x^2}{36} - \frac{y^2}{64} = 1$. Asymptotes: $y = \pm \frac{8}{6}x = \pm \frac{4}{3}x$. Eccentricity:

$$e = \frac{10}{6} = \frac{5}{3} \approx 1.67.$$

14 Eliminating by substitution

$t = x + 3$. Then $y = (x + 3)^2 + 2(x + 3) = x^2 + 6x + 9 + 2x + 6 = x^2 + 8x + 15$.

It is the parabola $y = x^2 + 8x + 15$, opening upward.

15 Trigonometric elimination

$\cos t = \frac{x}{4}$, $\sin t = \frac{y}{3}$. Using $\cos^2 t + \sin^2 t = 1$: $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

An ellipse with $a = 4$, $b = 3$, traced once counterclockwise starting at $(4, 0)$.

16 Find the eccentricity first

$c = ea = 0.6 \cdot 10 = 6$. Then $b^2 = a^2 - c^2 = 100 - 36 = 64$, so $b = 8$.

$$\frac{x^2}{100} + \frac{y^2}{64} = 1.$$

17 An ellipse in general form

$4(x^2 - 4x) + 9(y^2 + 2y) = 11$. Complete the squares:

$4(x - 2)^2 - 16 + 9(y + 1)^2 - 9 = 11$, so $4(x - 2)^2 + 9(y + 1)^2 = 36$.

Divide by 36: $\frac{(x - 2)^2}{9} + \frac{(y + 1)^2}{4} = 1$. Center $(2, -1)$, $a = 3$, $b = 2$, $c = \sqrt{5}$.

Foci $(2 \pm \sqrt{5}, -1)$; $e = \frac{\sqrt{5}}{3} \approx 0.75$.

18 A vertical hyperbola

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$9(y^2 - 4y) - 4(x^2 + 2x) = 4$. Complete the squares:

$9(y - 2)^2 - 36 - 4(x + 1)^2 + 4 = 4$, so $9(y - 2)^2 - 4(x + 1)^2 = 36$.

Standard form: $\frac{(y - 2)^2}{4} - \frac{(x + 1)^2}{9} = 1$. Center $(-1, 2)$, $a = 2$, $b = 3$, $c = \sqrt{13}$.

Vertices $(-1, 0)$ and $(-1, 4)$; foci $(-1, 2 \pm \sqrt{13})$; asymptotes $y - 2 = \pm \frac{2}{3}(x + 1)$.

19 A sideways parabola

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$y^2 - 6y = 8x - 25$. Add 9: $(y - 3)^2 = 8x - 16 = 8(x - 2)$.

So $4p = 8$, $p = 2$, vertex $(2, 3)$. Focus $(4, 3)$, directrix $x = 0$.

20 A planet's orbit

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The two extreme distances add up to the major axis: $2a = 90 + 150 = 240$, so $a = 120$.

The star is at distance c from the center, and the closest distance is $a - c = 90$, so $c = 30$.

$e = \frac{30}{120} = 0.25$. $b = \sqrt{120^2 - 30^2} = \sqrt{13\,500} = 30\sqrt{15} \approx 116.2$.

$a = 120$, $c = 30$, $e = 0.25$, $b \approx 116.2$ million miles.

21 Line and circle

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Substitute: $x^2 + (x + 1)^2 = 25$, so $2x^2 + 2x - 24 = 0$, that is $x^2 + x - 12 = 0$, or $(x + 4)(x - 3) = 0$.

$x = 3 \Rightarrow y = 4$; $x = -4 \Rightarrow y = -3$. The points are $(3, 4)$ and $(-4, -3)$. Both satisfy $x^2 + y^2 = 25$.

22 Projectile path



a. $t = \frac{x}{30}$, so $y = 5 + \frac{4}{3}x - \frac{4}{225}x^2$, a downward parabola.

b. The maximum is at $t = \frac{40}{32} = 1.25$ s: $y = 5 + 50 - 25 = 30$ ft (about 9.1 m).

c. Solve $y = 0$: $16t^2 - 40t - 5 = 0$, $t = \frac{40 + \sqrt{1920}}{32} \approx 2.619$ s. Then $x = 30t \approx 78.6$ ft (about 24.0 m).

23 Only part of a curve



$\sin^2 t = 1 - \cos^2 t$, so $y = 1 - x^2$.

But $x = \cos t$ only takes values in $[-1, 1]$, and y in $[0, 1]$. Only the arc of the parabola $y = 1 - x^2$ between $(-1, 0)$ and $(1, 0)$ is drawn, traced back and forth as t increases.

24 A hyperbola with sec and tan



$\frac{x}{2} = \sec t$ and $\frac{y}{3} = \tan t$. Since $\sec^2 t - \tan^2 t = 1$: $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

Here $a = 2$, $b = 3$, so the asymptotes are $y = \pm \frac{3}{2}x$.



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