

Conic Sections and Parametric Equations: math lesson, Grade 12 – download the PDF

Slice a cone with a flat plane and four famous curves appear: circles, ellipses, parabolas and hyperbolas. In Grade 12 you will describe each one with an equation, find its key points, measure how “stretched” it is with eccentricity, and then draw curves in a new way, with a parameter that moves a point along the path.

1. Where conic sections come from

Take a double cone. A plane that cuts straight across (perpendicular to the axis) gives a **circle**. Tilt the plane a little and you get an **ellipse**. Tilt it until it is parallel to the side of the cone and you get a **parabola**. Tilt it even more so that it cuts both halves of the cone and you get a **hyperbola** with two branches.

Every conic can also be defined by distances in the coordinate plane, and that is the definition we use to build equations.

2. Circles and general form

D Circle

A circle is the set of all points at a fixed distance r (the radius) from a fixed point (h, k) (the center). Its standard form is

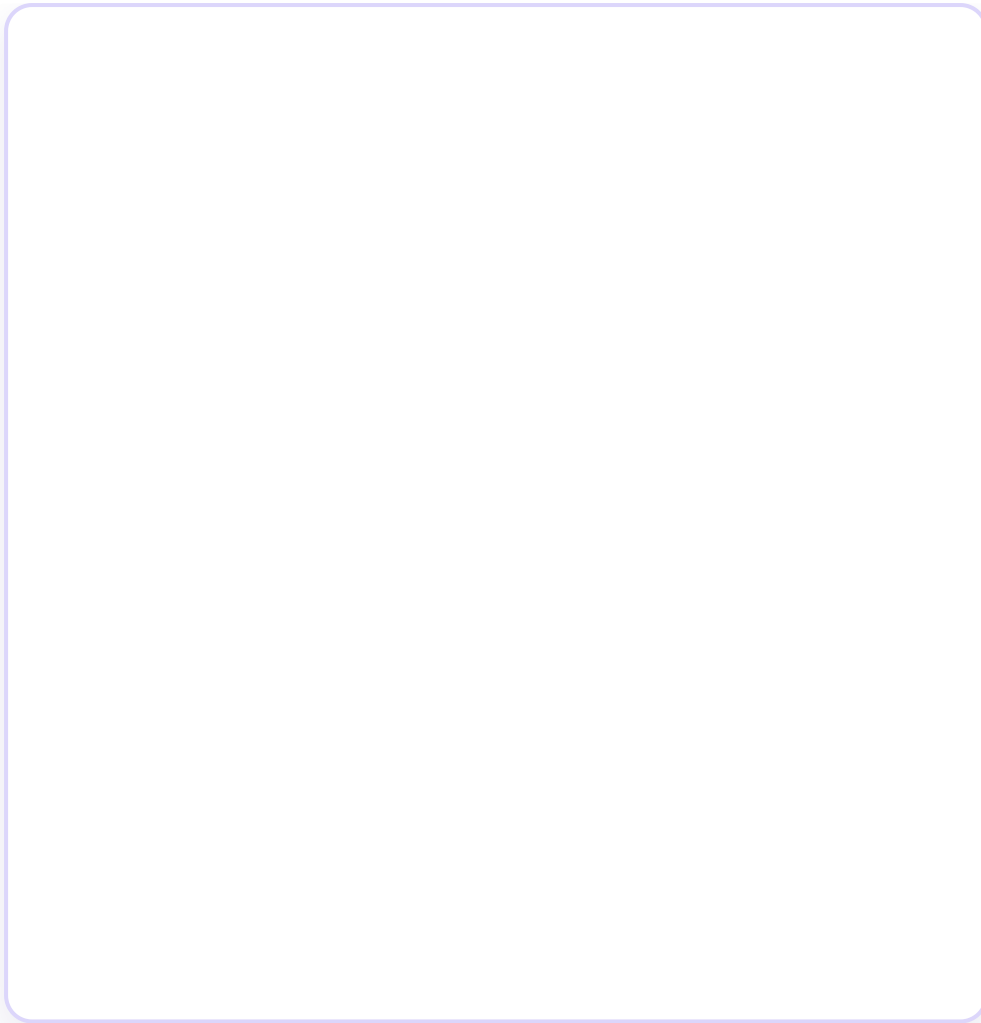
$$(x - h)^2 + (y - k)^2 = r^2.$$

Expanding the squares gives the **general form** $x^2 + y^2 + Dx + Ey + F = 0$. To go back to the standard form you **complete the square** in x and in y .

E Example 1: from general form to center and radius

Find the center and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

Group terms: $(x^2 - 6x) + (y^2 + 4y) = 12$. Add **9** and **4** to both sides:
 $(x - 3)^2 + (y + 2)^2 = 25$. The center is $(3, -2)$ and the radius is **5**.



Conic sections grade 12: circle with center $(3, -2)$ and radius 5 drawn on a coordinate grid with one radius to the point $(7, 1)$

3. Parabolas

Parabola

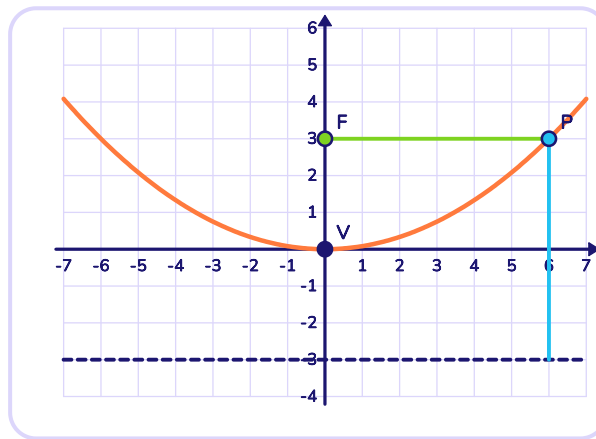
A parabola is the set of points that are the same distance from a fixed point, the **focus**, and from a fixed line, the **directrix**. The point halfway between them is the **vertex**.

P**Standard forms (vertex at (h, k) , focal distance p)**

Opens up or down: $(x - h)^2 = 4p(y - k)$, focus $(h, k + p)$, directrix $y = k - p$.

Opens right or left: $(y - k)^2 = 4p(x - h)$, focus $(h + p, k)$, directrix $x = h - p$.

If $p > 0$ it opens up (or right); if $p < 0$ it opens down (or left).

**E****Example 2: reading a parabola**

For $x^2 = 12y$: $4p = 12$, so $p = 3$. The vertex is $(0, 0)$, the focus is $(0, 3)$ and the directrix is $y = -3$. The point $P(6, 3)$ is on the curve because $6^2 = 12 \cdot 3$; its distance to the focus is **6** and its distance to the directrix is $3 - (-3) = 6$.

!**Zyro's tip**

On Earth I noticed that satellite dishes are parabolas. Every incoming signal bounces to the focus, so the receiver sits exactly there.

4. Ellipses

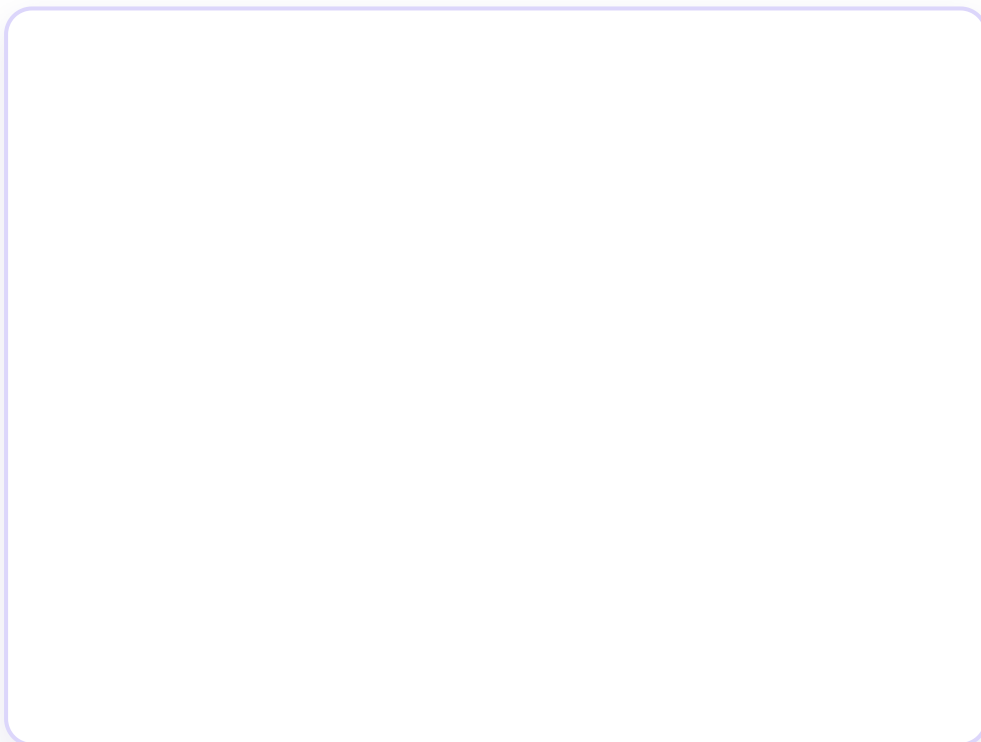
D Ellipse

An ellipse is the set of points whose distances to two fixed points, the **foci**, have a constant sum $2a$.

P Standard form (center at the origin, $a > b > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

has vertices $(\pm a, 0)$, co-vertices $(0, \pm b)$ and foci $(\pm c, 0)$ where $c^2 = a^2 - b^2$. If the larger denominator sits under y^2 , the long axis is vertical. With center (h, k) , replace x by $x - h$ and y by $y - k$.



Conic sections grade 12: ellipse $x^2/25 + y^2/9 = 1$ with foci at $(4, 0)$ and $(-4, 0)$ and a point whose two focal distances are 5 each

E**Example 3: an ellipse**

For $\frac{x^2}{25} + \frac{y^2}{9} = 1$: $a = 5$, $b = 3$, $c = \sqrt{25 - 9} = 4$. Vertices $(\pm 5, 0)$, co-vertices $(0, \pm 3)$, foci $(\pm 4, 0)$. Check with $P(0, 3)$: its distance to each focus is $\sqrt{16 + 9} = 5$, and $5 + 5 = 10 = 2a$.

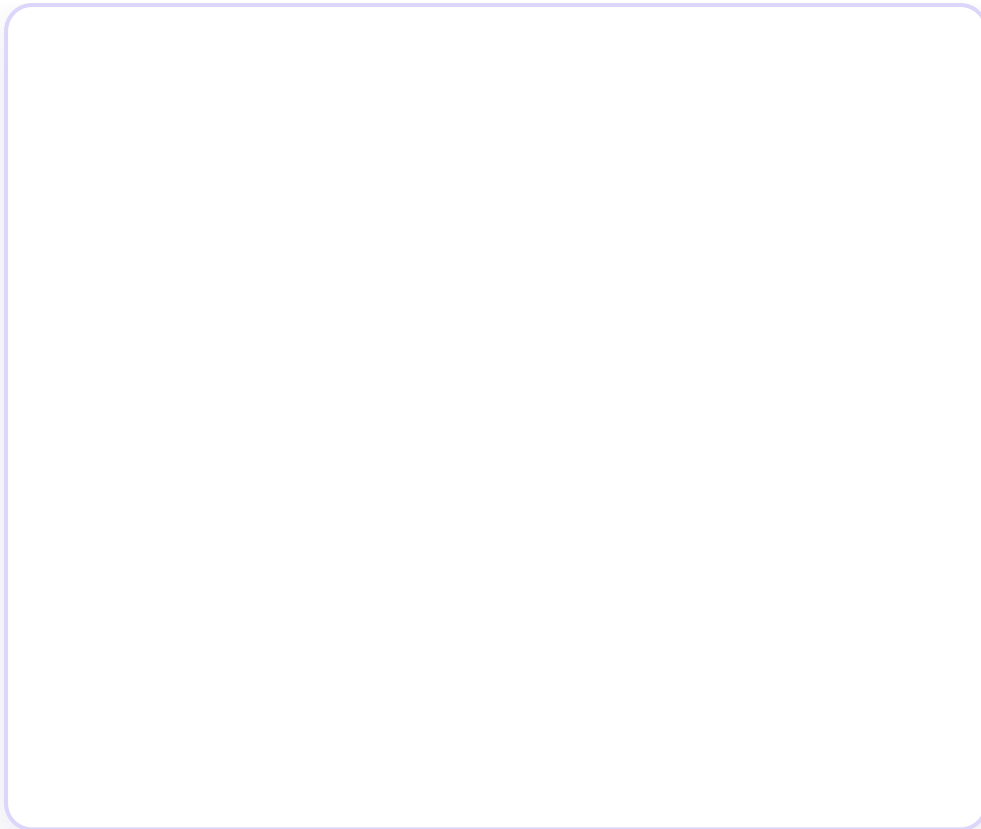
5. Hyperbolas**D****Hyperbola**

A hyperbola is the set of points for which the *difference* of the distances to two foci is constant, equal to $2a$ in absolute value.

P**Standard form (center at the origin)**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

has vertices $(\pm a, 0)$, foci $(\pm c, 0)$ with $c^2 = a^2 + b^2$, and asymptotes $y = \pm \frac{b}{a}x$. For $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ the branches open up and down, and the asymptotes are $y = \pm \frac{a}{b}x$.



Conic sections grade 12: hyperbola x^2 over 16 minus y^2 over 9 equals 1 with vertices at (4, 0) and (-4, 0), foci at (5, 0) and (-5, 0) and dashed asymptotes

E

Example 4: a hyperbola

For $\frac{x^2}{16} - \frac{y^2}{9} = 1$: $a = 4$, $b = 3$, $c = \sqrt{16 + 9} = 5$. Vertices $(\pm 4, 0)$, foci $(\pm 5, 0)$, asymptotes $y = \pm \frac{3}{4}x$.



Common mistake

For an ellipse $c^2 = a^2 - b^2$, but for a hyperbola $c^2 = a^2 + b^2$. Also, in a hyperbola a is not necessarily the larger number: it is the one under the *positive* term.

6. Eccentricity

D Eccentricity

For an ellipse or a hyperbola, the eccentricity is $e = \frac{c}{a}$. It measures how far the foci are from the center compared with the size of the curve.

Conic	Eccentricity	Shape
Circle	$e = 0$	foci merge into the center
Ellipse	$0 < e < 1$	closed oval, flatter as e grows
Parabola	$e = 1$	one open branch
Hyperbola	$e > 1$	two open branches

For $\frac{x^2}{25} + \frac{y^2}{9} = 1$, $e = \frac{4}{5} = 0.8$ (fairly stretched). For $\frac{x^2}{16} - \frac{y^2}{9} = 1$, $e = \frac{5}{4} = 1.25$.

7. Parametric equations

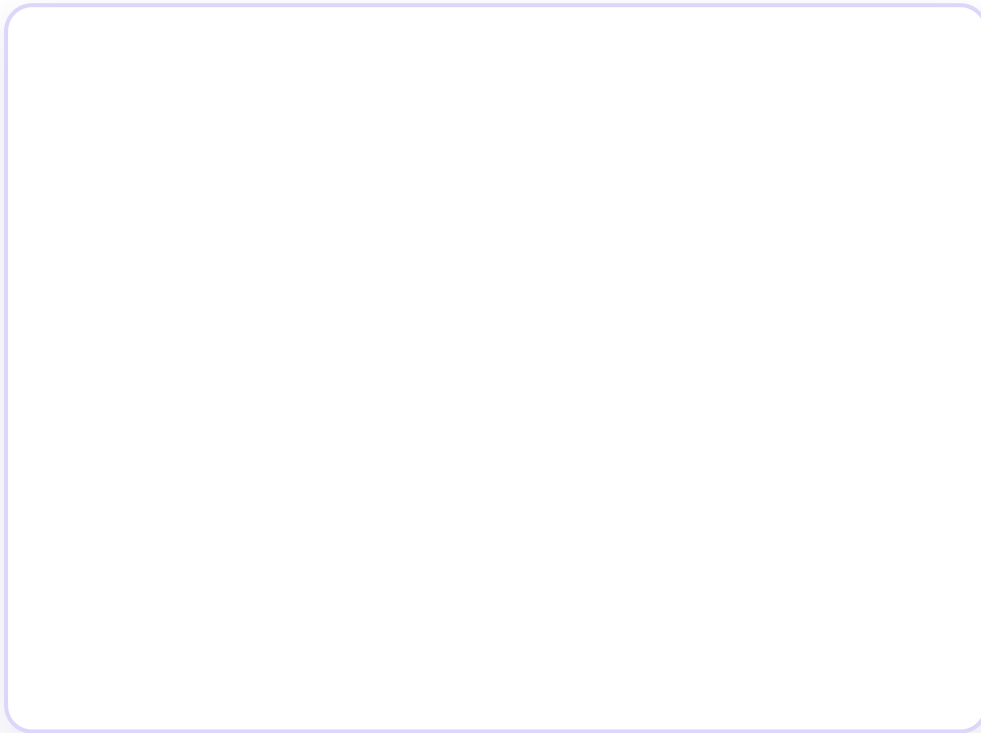
D Parametric equations

A curve can be given by $x = f(t)$ and $y = g(t)$, where the **parameter** t (often time or an angle) runs over an interval. Each value of t gives one point (x, y) , and the curve shows the *path* and the *direction of motion*.

P**Useful parametrizations**

Circle: $x = h + r \cos t$, $y = k + r \sin t$. Ellipse: $x = h + a \cos t$, $y = k + b \sin t$.

Hyperbola: $x = a \sec t$, $y = b \tan t$.



Conic sections grade 12: parametric path x equals $30t$ and y equals 5 plus $40t$ minus $16t$ squared of a thrown ball, with points marked every half second

E**Example 5: a thrown ball**

A ball follows $x = 30t$, $y = 5 + 40t - 16t^2$ (feet, seconds). At $t = 1$: $(30, 29)$. The height is largest when $t = \frac{40}{32} = 1.25$ s, where $y = 5 + 50 - 25 = 30$ ft (about 9.1 m).

8. Eliminating the parameter

M Method

1. If one equation is easy to solve for t , solve it and substitute into the other.
2. If the equations use $\cos t$ and $\sin t$ (or $\sec t$ and $\tan t$), isolate those functions and use $\cos^2 t + \sin^2 t = 1$ (or $\sec^2 t - \tan^2 t = 1$).
3. Look at the range of t : it may restrict which part of the curve is drawn.

E Example 6: algebraic elimination

$x = 2t + 1$, $y = t^2 - 3$. Then $t = \frac{x-1}{2}$, so $y = \frac{(x-1)^2}{4} - 3$, a parabola opening upward with vertex $(1, -3)$.

E Example 7: trigonometric elimination

$x = 3 \cos t$, $y = 2 \sin t$. Then $\cos t = \frac{x}{3}$ and $\sin t = \frac{y}{2}$, so $\frac{x^2}{9} + \frac{y^2}{4} = 1$, an ellipse traced once as t goes from 0 to 2π .

⚠ Watch the domain

$x = \cos t$, $y = \sin^2 t$ gives $y = 1 - x^2$, but only for $-1 \leq x \leq 1$: the curve is an arc, not the whole parabola.

Key takeaways

- Circle: $(x - h)^2 + (y - k)^2 = r^2$; complete the square to find the center and radius.
- Parabola: $(x - h)^2 = 4p(y - k)$ or $(y - k)^2 = 4p(x - h)$; focus at distance $|p|$ from the vertex, directrix on the other side.
- Ellipse: $c^2 = a^2 - b^2$; hyperbola: $c^2 = a^2 + b^2$ with asymptotes $y = \pm \frac{b}{a}x$.
- Eccentricity $e = c/a$: **0** for a circle, **< 1** ellipse, **1** parabola, **> 1** hyperbola.
- Parametric equations give a point for each t ; eliminate t by substitution or by an identity, and check the domain.



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