

Congruent Triangles: math lesson, Grade 10 – download the PDF

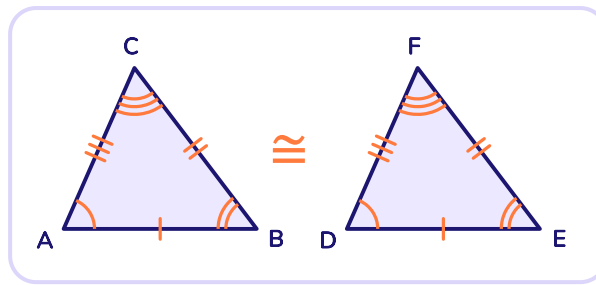
Two figures are congruent when one fits exactly on top of the other. In this chapter you will learn how to prove that two triangles are congruent with as few measurements as possible, and how to use that fact to uncover equal lengths and angles that nobody gave you.

1. Congruent triangles and corresponding parts

D Congruent triangles

Two triangles are **congruent** when you can move one onto the other with a slide, a turn, or a flip, so that they match perfectly. Then all three pairs of **corresponding sides** and all three pairs of **corresponding angles** are equal. We write $\triangle ABC \cong \triangle DEF$.

The order of the letters tells you which vertices match: in $\triangle ABC \cong \triangle DEF$, vertex **A** matches **D**, **B** matches **E**, and **C** matches **F**. So $AB = DE$, $BC = EF$, $CA = FD$, and $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$. In figures, identical tick marks show equal sides and identical arcs show equal angles.



Watch the letter order

Writing $\triangle ABC \cong \triangle EDF$ says something different from $\triangle ABC \cong \triangle DEF$. Always list matching vertices in the same position.

Checking six pairs of measurements every time would be exhausting. The good news: three well-chosen pairs are enough. The next sections show which ones.

2. The angle sum and exterior angle theorems



Triangle Angle Sum Theorem

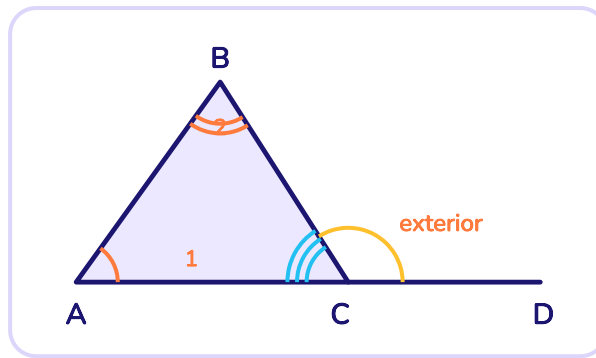
The three interior angles of any triangle add up to 180° .



Exterior Angle Theorem

An exterior angle of a triangle equals the sum of the two **remote interior angles**, the two interior angles that are not next to it.

Why does it work? The exterior angle at C and the interior angle at C form a straight line, so together they make 180° . The three interior angles also make 180° . Remove the interior angle at C from both sentences and you are left with exterior angle = $\angle A + \angle B$.



E Example 1: angle sum

A triangle has angles x° , $(2x + 5)^\circ$ and $(3x - 5)^\circ$. Find each angle.

The sum is $x + 2x + 5 + 3x - 5 = 6x$, and it must equal 180. So $x = 30$. The angles are 30° , 65° and 85° , and indeed $30 + 65 + 85 = 180$.

E Example 2: exterior angle

In a triangle, the two remote interior angles measure 48° and 67° . Find the exterior angle.

Exterior angle = $48^\circ + 67^\circ = 115^\circ$. Check: the interior angle next to it is $180 - 48 - 67 = 65^\circ$, and $65 + 115 = 180$.

3. The SSS and SAS criteria

P SSS (Side-Side-Side)

If three sides of one triangle are equal to three sides of another triangle, the triangles are congruent.

P**SAS (Side-Angle-Side)**

If two sides and the **included angle** (the angle between those two sides) of one triangle equal the matching parts of another, the triangles are congruent.

M**How to choose a criterion**

1. Mark everything you know: given information, vertical angles, and any side shared by both triangles (a shared side equals itself).
2. Count the matching pairs and note where the angles sit.
3. Name the criterion that fits, then write the congruence with the vertices in matching order.

E**Example 3: SSS**

In $\triangle ABC$, $AB = 7$ cm, $BC = 9$ cm, $CA = 11$ cm. In $\triangle DEF$, $DE = 7$ cm, $EF = 9$ cm, $FD = 11$ cm. Are they congruent?

The three pairs of sides are equal: $AB = DE$, $BC = EF$, $CA = FD$. By SSS, $\triangle ABC \cong \triangle DEF$. Because they are congruent, $\angle B = \angle E$ too, even though no angle was measured.

**Included means in between**

For SAS the angle must sit between the two known sides. If the angle is elsewhere, SAS does not apply.

4. ASA and AAS, and the two traps

P

ASA (Angle-Side-Angle)

Two angles and the **included side** of one triangle equal the matching parts of another: the triangles are congruent.

P

AAS (Angle-Angle-Side)

Two angles and a **non-included side** of one triangle equal the matching parts of another: the triangles are congruent. (This works because the third angles must be equal too, by the angle sum theorem, which turns AAS into ASA.)

Matching parts	Name	Proves congruence?
3 sides	SSS	Yes
2 sides and the angle between them	SAS	Yes
2 angles and the side between them	ASA	Yes
2 angles and a side not between them	AAS	Yes
3 angles	AAA	No: same shape, size may differ
2 sides and an angle not between them	SSA	No: two different triangles may fit



The two traps

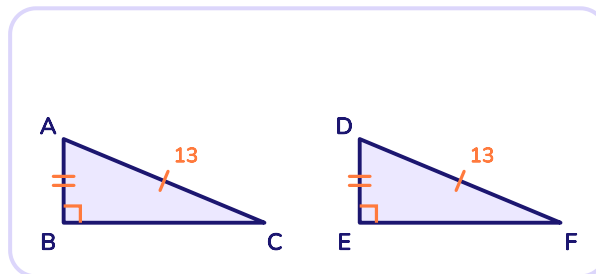
AAA only tells you the triangles have the same shape (a small and a large triangle can have the same angles). SSA is ambiguous: with the same two sides and a non-included angle, you can sometimes draw two different triangles.

5. The Hypotenuse-Leg theorem

P HL (Hypotenuse-Leg)

If the hypotenuse and one leg of a right triangle equal the hypotenuse and one leg of another right triangle, the triangles are congruent.

HL only works for **right** triangles. The reason is the Pythagorean theorem: once you know the hypotenuse and one leg, the other leg is forced, so all three sides match and SSS applies.



E Example 4: HL

$\triangle ABC$ is right at B and $\triangle DEF$ is right at E . Both hypotenuses measure **13** in and $AB = DE = 5$ in. Are they congruent?

Both are right triangles with equal hypotenuses and one pair of equal legs, so $\triangle ABC \cong \triangle DEF$ by HL. The other leg is $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$ in in both triangles (about **30.5** cm).

6. CPCTC and writing proofs

D CPCTC

CPCTC stands for **Corresponding Parts of Congruent Triangles are Congruent**. Once you have proved two triangles congruent, every pair of corresponding sides or angles is equal.

M Proof recipe

1. Prove two triangles congruent using SSS, SAS, ASA, AAS, or HL.
2. Write the congruence statement with vertices in matching order.
3. Use CPCTC to conclude the pair of sides or angles you wanted.

E**Example 5: a two-column proof**

Given: segments $\overline{AB}$ and $\overline{CD}$ cross at M , and M is the midpoint of both. Prove:
 $AC = BD$.

Statement	Reason
$AM = MB$ and $CM = MD$	Definition of midpoint
$\angle AMC = \angle BMD$	Vertical angles are equal
$\triangle AMC \cong \triangle BMD$	SAS (steps 1 and 2)
$AC = BD$	CPCTC

!**Zyro's tip**

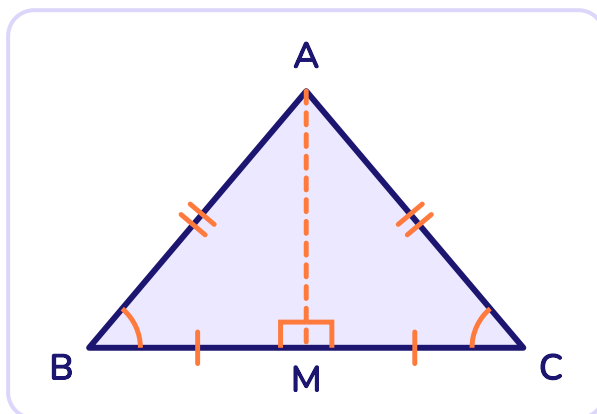
On my planet we say: first the triangles, then the parts. Never use CPCTC before you have finished proving the congruence!

7. Isosceles and equilateral triangles**P****Isosceles Triangle Theorem**

If two sides of a triangle are equal, the angles opposite them (the base angles) are equal. The converse is also true: if two angles are equal, the sides opposite them are equal.

P Equilateral triangles

A triangle is equilateral exactly when it is equiangular. Each angle then measures 60° .



Here is the idea behind the first theorem. In the figure, $AB = AC$ and M is the midpoint of $\overline{BC}$. The triangles ABM and ACM have three pairs of equal sides ($AB = AC$, $BM = CM$, and the shared side AM), so they are congruent by SSS. By CPCTC, $\angle B = \angle C$. Also $\angle AMB = \angle AMC$, and these two angles make a straight angle, so each is 90° : the median from the apex is also an altitude.

E Example 6: base angles

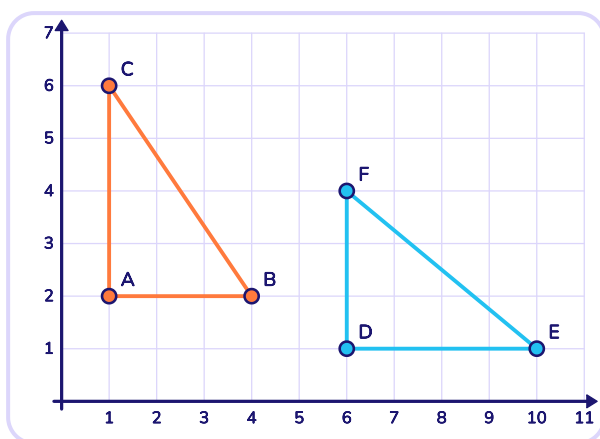
An isosceles triangle has an apex angle of 40° . Find the base angles.

The base angles are equal and the three angles add up to 180° , so each base angle is $(180 - 40) \div 2 = 70^\circ$.

8. Congruence in the coordinate plane

On a grid you can prove congruence with the distance formula: the distance between (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Compute the three side lengths of each triangle and compare them: if they match, SSS gives congruence. Rigid motions help too: a translation adds the same numbers to every point, a reflection across the x -axis changes (x, y) to $(x, -y)$, and a

quarter turn about the origin sends (x, y) to $(-y, x)$. A rigid motion always produces a congruent triangle.



E Example 7: SSS on a grid

Let $A(1, 2)$, $B(4, 2)$, $C(1, 6)$ and $D(6, 1)$, $E(10, 1)$, $F(6, 4)$. Are the triangles congruent?

$AB = 3$, $AC = 4$, $BC = \sqrt{3^2 + 4^2} = 5$. Also $DE = 4$, $DF = 3$,
 $EF = \sqrt{4^2 + 3^2} = 5$. The sides $3, 5, 4$ of $\triangle ABC$ (in the order AB, BC, CA) match
 DF, FE, ED , so $\triangle ABC \cong \triangle DFE$ by SSS.

Key takeaways

- The interior angles of a triangle add up to **180°** ; an exterior angle equals the sum of the two remote interior angles.
- Four criteria prove congruence: SSS, SAS, ASA, AAS. For right triangles, HL also works.
- AAA and SSA do **not** prove congruence.
- Write the congruence with vertices in matching order, then use CPCTC to get equal sides and angles.
- Base angles of an isosceles triangle are equal; an equilateral triangle has three **60°** angles.
- In the coordinate plane, compare the three side lengths with the distance formula.



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