

Complex Numbers: practice solutions, Grade 11 – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Square roots of negatives



a. $\sqrt{-36} = i\sqrt{36} = 6i.$

b. $\sqrt{-50} = i\sqrt{25 \cdot 2} = 5i\sqrt{2}.$

c. $-\sqrt{-81} = -9i.$

d. $\sqrt{-12} = i\sqrt{4 \cdot 3} = 2i\sqrt{3}.$

2 Powers of i



Divide each exponent by 4 and keep the remainder.

a. $15 = 4 \cdot 3 + 3$, so $i^{15} = i^3 = -i.$

b. $22 = 4 \cdot 5 + 2$, so $i^{22} = i^2 = -1.$

c. $40 = 4 \cdot 10 + 0$, so $i^{40} = 1.$

d. $101 = 4 \cdot 25 + 1$, so $i^{101} = i.$

3 Add and subtract



- a. Real parts: $5 + 3 = 8$. Imaginary parts: $2 - 7 = -5$. Answer: $8 - 5i$.
- b. $4 - 9 = -5$ and $-1 - 3 = -4$. Answer: $-5 - 4i$.
- c. $-2 - (-2) = 0$ and $6 - (-1) = 7$. Answer: $7i$.

4 Multiply and distribute



- a. $3i(4 - 5i) = 12i - 15i^2 = 12i + 15 = 15 + 12i$.
- b. $(1 + 3i)(2 - i) = 2 - i + 6i - 3i^2 = 2 + 5i + 3 = 5 + 5i$.

5 Conjugates



- a. The conjugate of $7 - 3i$ is $7 + 3i$.
- b. $-4i = 0 - 4i$, so its conjugate is $4i$.
- c. $9 = 9 + 0i$, so its conjugate is 9 itself.

Product: $(7 - 3i)(7 + 3i) = 7^2 + 3^2 = 49 + 9 = 58$.

6 Reading the complex plane



A point (a, b) represents $a + bi$.

- $A(2, 3)$: $2 + 3i$, real part 2 , imaginary part 3 .
- $B(-3, 1)$: $-3 + i$, real part -3 , imaginary part 1 .
- $C(-2, -2)$: $-2 - 2i$, real part -2 , imaginary part -2 .
- $D(4, -1)$: $4 - i$, real part 4 , imaginary part -1 .

7 Absolute values



a. $\sqrt{9 + 16} = \sqrt{25} = 5.$

b. $\sqrt{25 + 144} = \sqrt{169} = 13.$

c. $|-8i| = \sqrt{0 + 64} = 8.$

d. $\sqrt{36 + 64} = \sqrt{100} = 10.$

8 Matching parts



Two complex numbers are equal when their real parts match and their imaginary parts match.

Real parts: $x + 2 = 7$, so $x = 5$.

Imaginary parts: $3y - 1 = 5$, so $3y = 6$ and $y = 2$.

9 Products and squares



a. $12 + 15i - 8i - 10i^2 = 12 + 7i + 10 = 22 + 7i.$

b. $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i.$

c. $(2 - 3i)^2 = 4 - 12i + 9i^2 = 4 - 12i - 9 = -5 - 12i.$

10 Dividing complex numbers



a. Multiply by the conjugate $2 + i$:

$$\frac{(8 + i)(2 + i)}{(2 - i)(2 + i)} = \frac{16 + 8i + 2i + i^2}{4 + 1} = \frac{15 + 10i}{5} = 3 + 2i.$$

b. Multiply by $1 - i$: $\frac{6(1 - i)}{(1 + i)(1 - i)} = \frac{6 - 6i}{2} = 3 - 3i.$

11 Pure imaginary solutions



a. $x^2 = -49$, so $x = \pm 7i$.

b. $x^2 = -20$, so $x = \pm i\sqrt{20} = \pm 2i\sqrt{5}$.

c. $2x^2 = -18$, so $x^2 = -9$ and $x = \pm 3i$.

12 Quadratic formula



The discriminant is $(-6)^2 - 4(1)(13) = 36 - 52 = -16$.

$$x = \frac{6 \pm \sqrt{-16}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i.$$

Check: the sum of the roots is **6** and their product is $3^2 + 2^2 = 13$, as expected.

13 Circuit impedance



a. $Z = (3 + 4i) + (2 - i) = 5 + 3i$ ohms.

b. $|Z| = \sqrt{25 + 9} = \sqrt{34} \approx 5.83$ ohms.

14 True or false?



a. **False.** The product rule for radicals does not apply to negative numbers. Correctly,

$$\sqrt{-4} \cdot \sqrt{-9} = 2i \cdot 3i = 6i^2 = -6.$$

b. **True** for these numbers. $z + w = 3 - i$, so $\overline{z + w} = 3 + i$. Also

$$\bar{z} + \bar{w} = (2 - 3i) + (1 + 4i) = 3 + i. \text{ The two results agree.}$$

15 Distance in the plane



The distance is the absolute value of the difference: $(2 + i) - (-1 + 5i) = 3 - 4i$.

$$|3 - 4i| = \sqrt{9 + 16} = 5. \text{ They are } \mathbf{5 \text{ yards}} \text{ apart (about } \mathbf{4.6} \text{ meters).}$$

16 Complex roots with coefficients



Discriminant: $16 - 36 = -20$.

$$x = \frac{-4 \pm \sqrt{-20}}{2} = \frac{-4 \pm 2i\sqrt{5}}{2} = -2 \pm i\sqrt{5}.$$

Check: the sum is $-4 = -\frac{b}{a}$ and the product is $(-2)^2 + (\sqrt{5})^2 = 4 + 5 = 9 = \frac{c}{a}$.

17 Build the equation



The sum of the roots is 4 and their product is $(2 + 3i)(2 - 3i) = 4 + 9 = 13$.

An equation with sum S and product P is $x^2 - Sx + P = 0$, so $x^2 - 4x + 13 = 0$.

Check with the formula: the discriminant is $16 - 52 = -36$, and $x = \frac{4 \pm 6i}{2} = 2 \pm 3i$.

18 Higher powers



a. $(2 + i)^2 = 4 + 4i + i^2 = 3 + 4i$. Then

$$(3 + 4i)(2 + i) = 6 + 3i + 8i + 4i^2 = 2 + 11i.$$

b. $(1 - i)^2 = 1 - 2i + i^2 = -2i$. Then $(1 - i)^4 = (-2i)^2 = 4i^2 = -4$.

19 A square root of a complex number



$(2 + 3i)^2 = 4 + 12i + 9i^2 = 4 + 12i - 9 = -5 + 12i$. So $2 + 3i$ is a square root of $-5 + 12i$.

Since $(-z)^2 = z^2$, the other square root is $-(2 + 3i) = -2 - 3i$.

20 Reciprocal with the conjugate



a. $\bar{z} = 8 + 15i$ and $z\bar{z} = 8^2 + 15^2 = 64 + 225 = 289$. Since $z\bar{z} = |z|^2$,
 $|z| = \sqrt{289} = 17$.

b. $\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{8 + 15i}{289} = \frac{8}{289} + \frac{15}{289}i$.

21 Quarter turns



One move: $i(4 + i) = 4i + i^2 = -1 + 4i$.

Two moves: $i(-1 + 4i) = -i + 4i^2 = -4 - i$.

Three moves: $i(-4 - i) = -4i - i^2 = 1 - 4i$.

Four moves: $i(1 - 4i) = i - 4i^2 = 4 + i$. The drone is back at its start, because $i^4 = 1$.



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