

Complex Numbers: math lesson, Grade 11 – download the PDF

Try solving $x^2 = -1$ with the numbers you know: every real number squared is zero or positive, so there is no solution. Mathematicians fixed this by inventing a new number, and it turned out to be one of the most useful ideas ever: electrical engineers, game designers, and signal analysts use it every day. In this chapter you will learn to calculate with **complex numbers**, solve every quadratic equation, and picture these numbers as points in a plane.

1. The imaginary unit i

D Imaginary unit

The **imaginary unit** is the number i defined by $i^2 = -1$. We also write $i = \sqrt{-1}$.

A **complex number** is a number that can be written in **standard form** $a + bi$, where a and b are real numbers. We call a the **real part** and b the **imaginary part**. For example, in $5 - 4i$ the real part is **5** and the imaginary part is **-4** (the number -4 , not $-4i$). If $b = 0$ the number is real, so every real number is also a complex number. If $a = 0$, the number bi is called **pure imaginary**. Two complex numbers are equal exactly when their real parts are equal and their imaginary parts are equal.

P**Square roots of negative numbers**

For any positive real number n , $\sqrt{-n} = i\sqrt{n}$.

E**Example 1**

Write $\sqrt{-36}$ and $\sqrt{-45}$ in terms of i .

$$\sqrt{-36} = i\sqrt{36} = 6i.$$

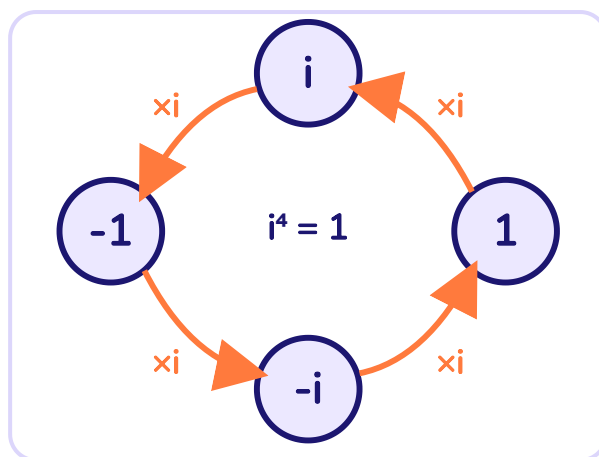
$$\sqrt{-45} = i\sqrt{45} = i\sqrt{9 \cdot 5} = 3i\sqrt{5}.$$

⚠**Watch out**

The rule $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ only works when a and b are not negative. Always rewrite each radical with i first: $\sqrt{-4} \cdot \sqrt{-9} = 2i \cdot 3i = 6i^2 = -6$, not 6.

2. Powers of i

Start multiplying by i again and again: $i^1 = i$, $i^2 = -1$, $i^3 = i^2 \cdot i = -i$, $i^4 = i^2 \cdot i^2 = 1$, and then $i^5 = i$ again. The pattern repeats every four steps.



M Method: power of i

1. Divide the exponent by **4** and keep only the remainder r .
2. Replace i^n by i^r , with $i^0 = 1$.
3. Use the cycle **1, i , -1 , $-i$** .

E Example 2

Simplify i^{58} and i^{103} .

$$58 = 4 \cdot 14 + 2, \text{ so } i^{58} = i^2 = -1.$$

$$103 = 4 \cdot 25 + 3, \text{ so } i^{103} = i^3 = -i.$$

3. Adding, subtracting, and multiplying

To **add or subtract** complex numbers, combine the real parts together and the imaginary parts together, just like combining like terms: $(a + bi) + (c + di) = (a + c) + (b + d)i$. To **multiply**, distribute (FOIL works well) and then replace i^2 by -1 .

E**Example 3**

Compute $(3 + 2i)(4 - 5i)$.

$$(3 + 2i)(4 - 5i) = 12 - 15i + 8i - 10i^2 = 12 - 7i - 10(-1) = 22 - 7i.$$

!**Zyro's tip**

On my home planet we say: “every time an i^2 shows up, it turns into a -1 ”. After that, the answer must have only one i in it, never i^2 .

4. Conjugates and division**D****Complex conjugate**

The **conjugate** of $z = a + bi$ is $\bar{z} = a - bi$: only the sign of the imaginary part changes.

The product of a complex number and its conjugate is always a non-negative real number: $(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2$. This is exactly what we need to divide: multiplying the top and the bottom of a fraction by the conjugate of the denominator removes i from the denominator.

M**Method: dividing complex numbers**

1. Write the conjugate of the denominator.
2. Multiply the numerator and the denominator by it.
3. Expand the numerator; the denominator becomes $a^2 + b^2$.
4. Write the answer in the form $a + bi$.

E**Example 4**

Write $\frac{5+i}{2-3i}$ in standard form.

$$\frac{5+i}{2-3i} = \frac{(5+i)(2+3i)}{(2-3i)(2+3i)} = \frac{10+15i+2i+3i^2}{4+9} = \frac{7+17i}{13} = \frac{7}{13} + \frac{17}{13}i.$$

5. Complex solutions of quadratics

For $ax^2 + bx + c = 0$, the quadratic formula gives $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. When the discriminant $b^2 - 4ac$ is negative, the radical is the square root of a negative number, and the equation now has solutions: two **complex** ones.

P**Conjugate pairs**

If a quadratic equation has real coefficients and a negative discriminant, it has exactly two non-real solutions, and they are complex conjugates of each other.

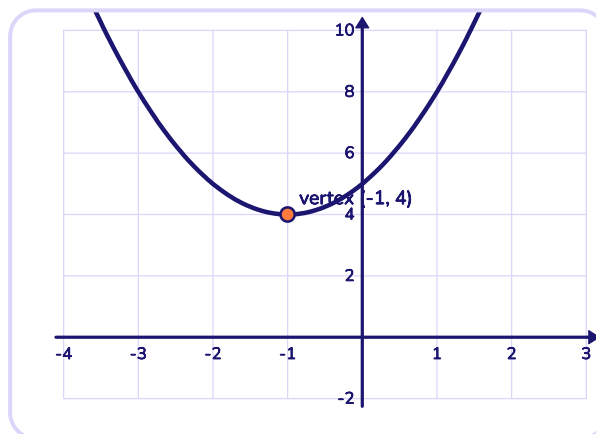
E**Example 5**

Solve $x^2 + 6x + 25 = 0$.

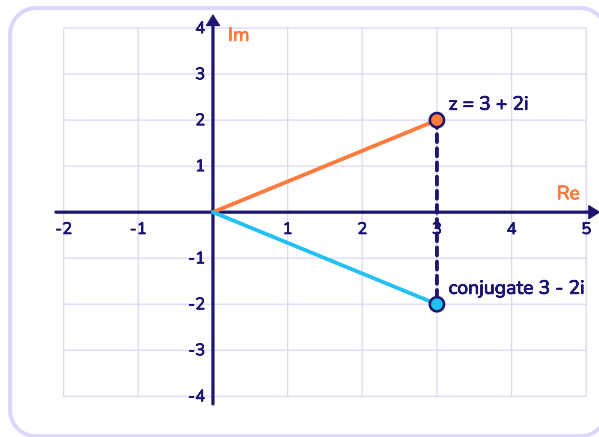
The discriminant is $6^2 - 4(1)(25) = 36 - 100 = -64 < 0$.

$$x = \frac{-6 \pm \sqrt{-64}}{2} = \frac{-6 \pm 8i}{2} = -3 \pm 4i.$$

Graphically, a negative discriminant means that the parabola never touches the x -axis. For instance, $y = x^2 + 2x + 5$ has its lowest point at $(-1, 4)$, so it stays above the axis. Its complex roots are $-1 \pm 2i$.

**6. The complex plane**

Every complex number $a + bi$ matches exactly one point (a, b) . In the **complex plane** the horizontal axis is the **real axis** and the vertical axis is the **imaginary axis**. The conjugate of a point is its mirror image across the real axis.



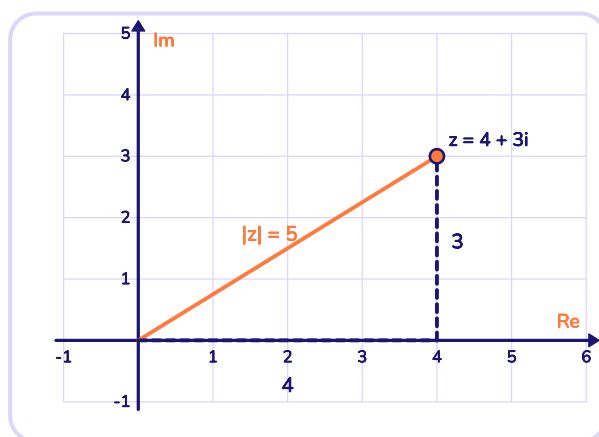
Adding complex numbers moves points like adding vectors. Multiplying by i turns a point a quarter turn (90°) counterclockwise around the origin: $i(a + bi) = -b + ai$. That is why the powers of i hop around the plane in a square.

7. Absolute value of a complex number

D Absolute value (modulus)

The **absolute value** of $z = a + bi$ is its distance from the origin in the complex plane:

$$|a + bi| = \sqrt{a^2 + b^2}$$



The formula comes from the Pythagorean theorem in the right triangle with legs $|a|$ and $|b|$. Some useful facts: $|z| = |\bar{z}|$, $z \cdot \bar{z} = |z|^2$, $|zw| = |z| |w|$, and the distance between two complex numbers z and w is $|z - w|$.

E**Example 6**

Find $|-5 + 12i|$, and the distance between $1 + 2i$ and $4 + 6i$.

$$|-5 + 12i| = \sqrt{25 + 144} = \sqrt{169} = 13.$$

The difference is $(4 + 6i) - (1 + 2i) = 3 + 4i$, so the distance is $\sqrt{9 + 16} = 5$ units.

Key takeaways

- $i^2 = -1$, and $\sqrt{-n} = i\sqrt{n}$ for $n > 0$. Rewrite radicals with i before multiplying.
- Powers of i repeat in a cycle of four: $i, -1, -i, 1$. Use the remainder when the exponent is divided by 4.
- Add and subtract the real and imaginary parts separately; multiply with FOIL and replace i^2 by -1 .
- The conjugate of $a + bi$ is $a - bi$, and $(a + bi)(a - bi) = a^2 + b^2$. Use it to divide.
- A quadratic with real coefficients and a negative discriminant has two conjugate complex solutions.
- $a + bi$ is the point (a, b) in the complex plane, and $|a + bi| = \sqrt{a^2 + b^2}$ is its distance from the origin.

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