

Circles: math lesson, Grade 10 – download the PDF

Wheels, clocks, pizzas, satellite dishes, and running tracks all hide the same shape. In this chapter you will learn the theorems that connect the lines, angles, arcs, and lengths of a circle, and you will place circles on the coordinate plane using their equations.

1. Chords, radii, and tangents

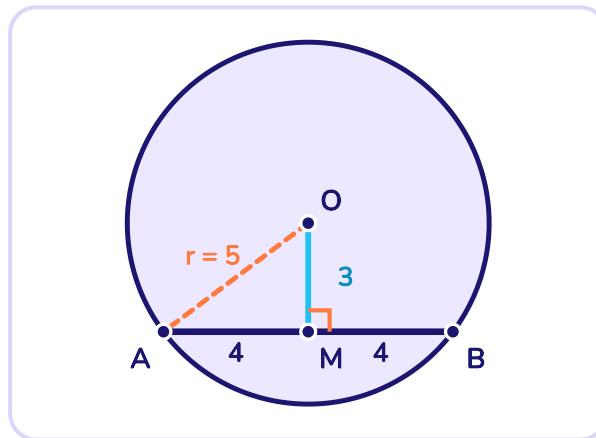
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Vocabulary

A **circle** is the set of all points in a plane at the same distance r (the **radius**) from a fixed point O (the **center**). A **chord** joins two points of the circle. A **diameter** is a chord through the center, so $d = 2r$. A **tangent** is a line that touches the circle at exactly one point, called the **point of tangency**. A **secant** is a line that meets the circle at two points.

P**Theorems about chords and tangents**

- A radius drawn to the point of tangency is perpendicular to the tangent line.
- If a line through the center is perpendicular to a chord, it bisects the chord (and its arc).
- In the same circle, congruent chords are the same distance from the center, and the diameter is the longest chord.
- Two tangent segments drawn from the same external point are congruent.

**E****Example 1: distance to a chord**

A circle has radius **10** cm and a chord of length **16** cm. How far is the chord from the center?

The perpendicular from O bisects the chord, so we get a right triangle with legs d and 8 and hypotenuse 10. Then $d^2 = 10^2 - 8^2 = 36$, so $d = 6$ cm.



Common mistake

Do not use the full chord length in the right triangle. Only *half* of the chord is a leg of the triangle.

2. Central and inscribed angles



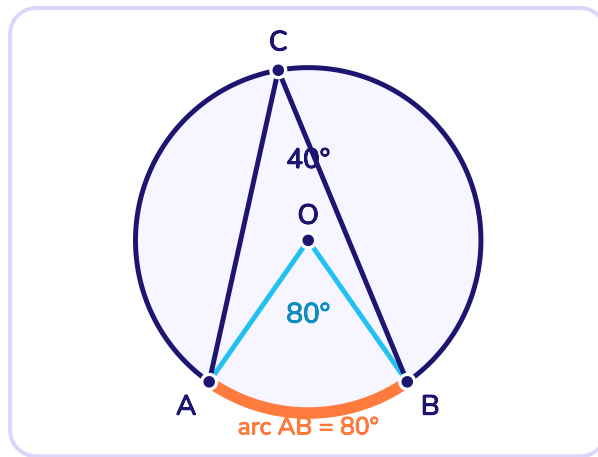
Central and inscribed angles

A **central angle** has its vertex at the center, and its measure equals the measure of its intercepted arc. An **inscribed angle** has its vertex on the circle, with both sides being chords.



Inscribed angle theorem

- An inscribed angle measures **half** of its intercepted arc: $m\angle ACB = \frac{1}{2} m\widehat{AB}$.
- Inscribed angles that intercept the same arc are congruent.
- An angle inscribed in a semicircle is a right angle.
- Opposite angles of a quadrilateral inscribed in a circle are supplementary (they add up to 180°).



E Example 2: using the inscribed angle theorem

An inscribed angle $\angle PQR$ intercepts an arc $\widehat{PR}$ of 130° . Then $m\angle PQR = 130^\circ \div 2 = 65^\circ$. If PR were a diameter, the arc would be 180° and the angle 90° .

3. Arc measure and arc length

The **measure** of an arc is an angle in degrees, but its **length** is a distance. A full circle has 360° and circumference $C = 2\pi r$, so an arc of θ° is the fraction $\frac{\theta}{360}$ of the circle.

P Arc length

$$\ell = \frac{\theta}{360} \cdot 2\pi r$$

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Example 3: length of an arc

A circle has radius 9 in. Find the length of a 70° arc.

$$\ell = \frac{70}{360} \cdot 2\pi \cdot 9 = \frac{70}{360} \cdot 18\pi = 3.5\pi \approx 11.0 \text{ in. In metric units, that is about } 27.9 \text{ cm, since } 1 \text{ in} = 2.54 \text{ cm.}$$

4. Sector area

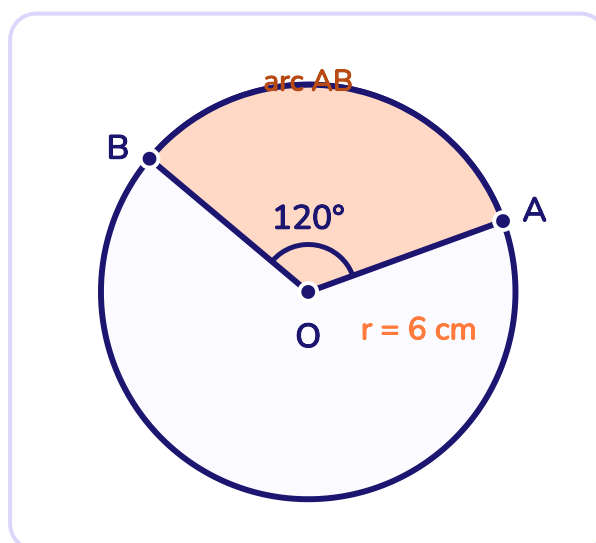
A **sector** is the region bounded by two radii and the arc between them. Just as with arcs, a sector with central angle θ is the fraction $\frac{\theta}{360}$ of the whole disk.

P

Sector area

$$A = \frac{\theta}{360} \cdot \pi r^2$$

The area of a **segment** (the region between a chord and its arc) is the sector area minus the area of the triangle formed by the two radii and the chord.



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Example 4: sector of a circle

For the sector in the figure, $r = 6$ cm and $\theta = 120^\circ$. Then

$A = \frac{120}{360} \cdot \pi \cdot 36 = 12\pi \approx 37.7 \text{ cm}^2$, and the arc length is $\frac{120}{360} \cdot 12\pi = 4\pi \approx 12.6$ cm.

5. Angles formed by secants and tangents

When two lines cut a circle, the angle between them depends on the arcs they intercept.

Where is the vertex?	Angle measure
On the circle (tangent and chord)	$\frac{1}{2}$ (intercepted arc)
Inside the circle (two chords or secants)	$\frac{1}{2}$ (arc + vertical arc)
Outside the circle (two secants, a tangent and a secant, or two tangents)	$\frac{1}{2}$ (far arc – near arc)

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Example 5: angle outside the circle

Two secants meet at a point outside a circle. They intercept a far arc of 118° and a near arc of 34° . The angle at the vertex measures $\frac{1}{2}(118 - 34) = 42^\circ$.



Zyro's tip

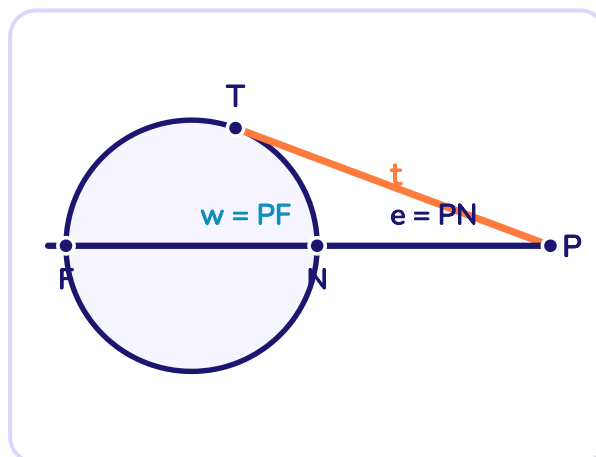
On my planet we say: “inside, add; outside, subtract.” Inside the circle you add the two arcs, outside you subtract the near arc from the far arc. In both cases, divide by 2!

6. Segment lengths in circles



Segment length theorems

- **Two chords:** if chords AB and CD meet at E , then $AE \cdot EB = CE \cdot ED$.
- **Two secants:** from an external point, (whole secant) $\times$ (external part) is the same for both secants.
- **Tangent and secant:** $t^2 = w \cdot e$, where t is the tangent length, w the whole secant, and e its external part.



E Example 6: tangent and secant

From a point P , a tangent has length **12** and a secant has external part **8**. Find the length of the chord inside the circle.

$12^2 = 8 \cdot w$, so $w = 144 \div 8 = 18$. The chord is $w - e = 18 - 8 = 10$ units long.

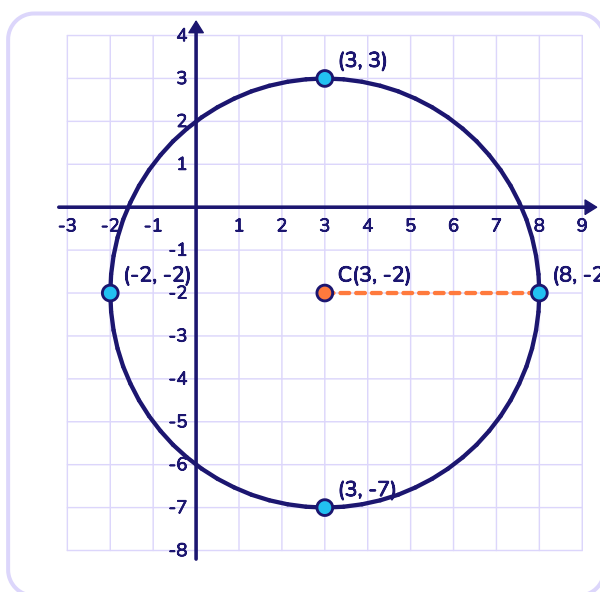
7. Equation of a circle

P Standard form

The circle with center (h, k) and radius r has equation

$$(x - h)^2 + (y - k)^2 = r^2.$$

This comes straight from the distance formula: every point (x, y) on the circle is at distance r from (h, k) .



M**Method: from general form to center and radius**

1. Group the x -terms and the y -terms, and move the constant to the right.
2. Complete the square in each group, adding the same amounts to both sides.
3. Read (h, k) and r^2 ; then take the square root to get r .

E**Example 7: completing the square**

Find the center and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

$(x^2 - 6x + 9) + (y^2 + 4y + 4) = 12 + 9 + 4$, so $(x - 3)^2 + (y + 2)^2 = 25$. The center is $(3, -2)$ and the radius is 5.

A**Common mistake**

The signs inside the parentheses are opposite to the center's coordinates: $(y + 2)^2$ means $k = -2$. Also, the right side is r^2 , not r .

8. Inscribed and circumscribed polygons**D****Inscribed and circumscribed**

A polygon is **inscribed** in a circle if all of its vertices lie on the circle; the circle is then **circumscribed** about the polygon. A circle is **inscribed** in a polygon if it is tangent to every side.

- The center of the circle circumscribed about a triangle (the **circumcenter**) is where the perpendicular bisectors of the sides meet.
- The center of the inscribed circle (the **incenter**) is where the angle bisectors meet.
- In a right triangle, the hypotenuse is a diameter of the circumscribed circle, so $R = \frac{c}{2}$.
- For a triangle with area A and semiperimeter s , the inscribed circle has radius $r = \frac{A}{s}$.
- A regular hexagon inscribed in a circle of radius R has side length R .

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Example 8: circles of a right triangle

A right triangle has legs **6** and **8**, so its hypotenuse is **10**. The circumscribed circle has radius $R = 5$. The area is **24** and the semiperimeter is **12**, so the inscribed circle has radius $r = 24 \div 12 = 2$.

Key takeaways

- A radius to a point of tangency is perpendicular to the tangent; tangent segments from one point are congruent.
- A perpendicular from the center bisects a chord.
- An inscribed angle is half its intercepted arc; opposite angles of an inscribed quadrilateral add up to 180° .
- Arc length: $\frac{\theta}{360} \cdot 2\pi r$. Sector area: $\frac{\theta}{360} \cdot \pi r^2$.
- Angles: on the circle, half the arc; inside, half the sum; outside, half the difference.
- Chords: $AE \cdot EB = CE \cdot ED$. Tangent and secant: $t^2 = w \cdot e$.
- Circle equation: $(x - h)^2 + (y - k)^2 = r^2$.



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