

Area, Surface Area, and Volume: math lesson, Grade 10 – download the PDF

How much paint covers a wall? How much water fills a tank? How much metal does a can need? Every one of these questions is about measuring a figure: **area** for flat regions, **surface area** for the outside of a solid, and **volume** for the space inside. In this chapter you will build a toolbox of formulas, learn why they work, and then use them to model real objects.

1. Area of polygons and composite figures

Area counts how many unit squares fit inside a flat region, so it is always written in *square units* (square inches, square meters, and so on). Here are the formulas you need.

Figure	Area
Rectangle (base b, height h)	$A = bh$
Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2}bh$
Trapezoid (bases b_1, b_2)	$A = \frac{1}{2}(b_1 + b_2)h$
Circle	$A = \pi r^2$
Regular polygon (apothem a, perimeter P)	$A = \frac{1}{2}aP$

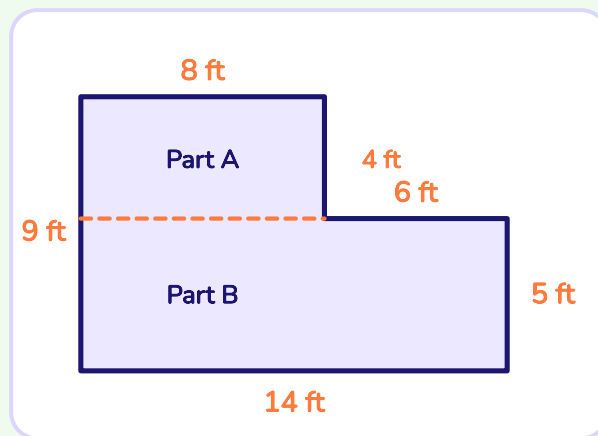
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Method: composite figures

1. Look for simple pieces: rectangles, triangles, trapezoids, circles or parts of circles.
2. Either **add** the pieces, or **subtract** a missing piece from a larger shape.
3. Find every missing length from the ones you are given.
4. Write the total with the correct square unit.

E**Example 1: an L-shaped floor**

The floor below is 14 ft by 9 ft with a 6 ft by 4 ft corner removed.



Subtract the missing corner from the full rectangle:

$14 \times 9 - 6 \times 4 = 126 - 24 = 102$. The area is **102 square feet**. Splitting along the dashed line gives the same answer: $14 \times 5 + 8 \times 4 = 70 + 32 = 102$.

**Watch your units**

One foot is 12 inches, but one *square* foot is $12 \times 12 = 144$ square inches. Likewise $1 \text{ m}^2 = 10,000 \text{ cm}^2$. Convert lengths before you multiply, or convert areas with the squared factor.

2. Prisms and cylinders

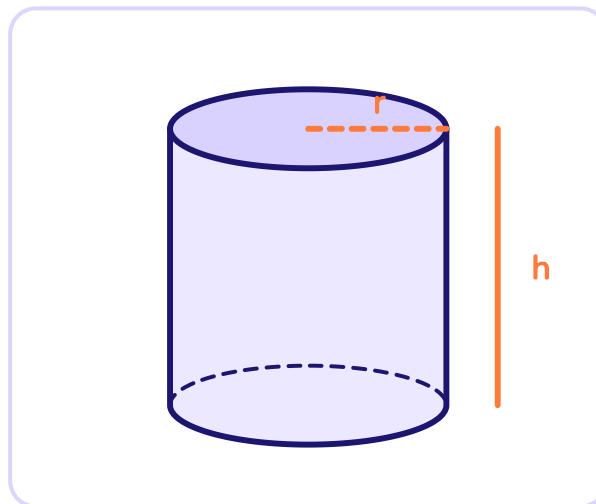
A **prism** has two parallel, congruent polygons as bases joined by rectangles (or parallelograms). A **cylinder** is the same idea with circles as bases. In both cases the solid is a stack of identical layers, so its volume is the area of one layer times the height.

P**Prisms and cylinders**

Let B be the area of a base, P its perimeter, and h the height of a right solid.

- Volume of a prism or a cylinder: $V = Bh$.
- Prism: $SA = 2B + Ph$, where Ph is the lateral area.
- Cylinder of radius r : $V = \pi r^2 h$ and $SA = 2\pi r^2 + 2\pi rh$.

The lateral surface of a cylinder unrolls into a rectangle whose width is the circumference $2\pi r$ and whose height is h ; that is why the lateral area is $2\pi rh$.

**E****Example 2: a cylinder**

A can has radius 3 in and height 8 in. Then $V = \pi \cdot 3^2 \cdot 8 = 72\pi \approx 226.19 \text{ in}^3$. Its total surface area is $2\pi \cdot 9 + 2\pi \cdot 3 \cdot 8 = 18\pi + 48\pi = 66\pi \approx 207.35 \text{ in}^2$.

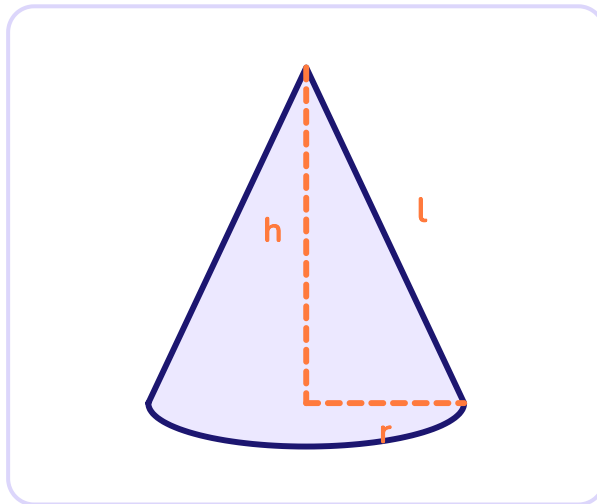
3. Pyramids and cones

A pyramid has one polygon as its base and triangular faces meeting at the apex; a cone has a circular base. The key fact is that a pyramid or cone holds exactly **one third** of the prism or cylinder with the same base and height.

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Pyramids and cones

- $V = \frac{1}{3}Bh$ for any pyramid, and $V = \frac{1}{3}\pi r^2 h$ for a cone.
- The **slant height** ℓ is the distance from the apex down a face to the edge of the base. For a cone, $\ell = \sqrt{r^2 + h^2}$.
- Cone: $SA = \pi r^2 + \pi r \ell$.
- Regular pyramid: $SA = B + \frac{1}{2}P\ell$.



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Example 3: a cone

A cone has radius 5 cm and height 12 cm. The height, radius and slant height form a right triangle, so $\ell = \sqrt{25 + 144} = 13$ cm. Then

$$V = \frac{1}{3}\pi \cdot 25 \cdot 12 = 100\pi \approx 314.16 \text{ cm}^3 \text{ and}$$

$$SA = 25\pi + \pi \cdot 5 \cdot 13 = 90\pi \approx 282.74 \text{ cm}^2.$$



Height is not slant height

Volume uses the vertical height h . Surface area uses the slant height ℓ . Mixing them up is the most common mistake in this chapter.

4. Spheres



Sphere

A sphere is the set of all points in space at the same distance r (the radius) from a center.

It has no flat faces or edges, yet its formulas are short.



Sphere

$V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$. A hemisphere has half the volume, and its curved surface has half the area (add πr^2 if the flat circle is included).

Notice that the surface area is exactly four times the area of a great circle (the circle through the center). Because volume involves r^3 , doubling the radius multiplies the volume by 8 but the surface area only by 4.

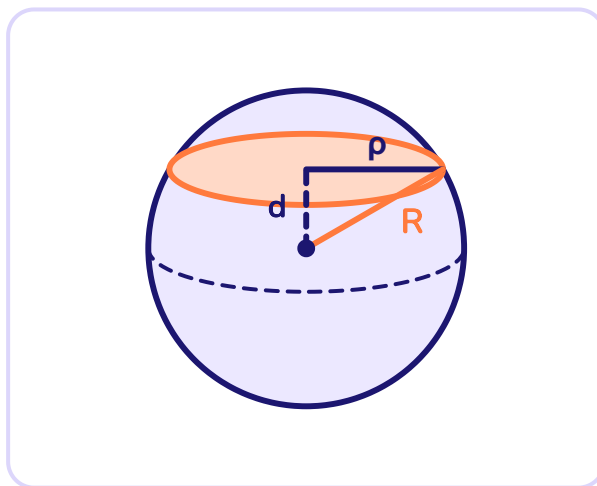
E**Example 4: a ball**

A ball has radius 6 cm: $V = \frac{4}{3}\pi \cdot 216 = 288\pi \approx 904.78 \text{ cm}^3$ and
 $SA = 4\pi \cdot 36 = 144\pi \approx 452.39 \text{ cm}^2$.

5. Cross sections of solids

A **cross section** is the shape you see when a plane slices through a solid. Knowing the shape lets you compute areas inside a solid.

- Cylinder: a plane parallel to the bases gives a circle; a plane through the axis gives a rectangle.
- Cone: parallel to the base gives a circle; through the apex and the base gives a triangle.
- Pyramid: parallel to the base gives a smaller copy of the base.
- Sphere: *every* plane section is a circle.
- Prism: parallel to the bases gives a copy of the base.



For a sphere of radius R cut by a plane at distance d from the center, the right triangle in the figure gives $p^2 = R^2 - d^2$ for the radius of the circle. For a pyramid or cone cut parallel to the base, the section is **similar** to the base: if the plane is a fraction k of the way from the apex to the base, lengths scale by k and areas by k^2 .

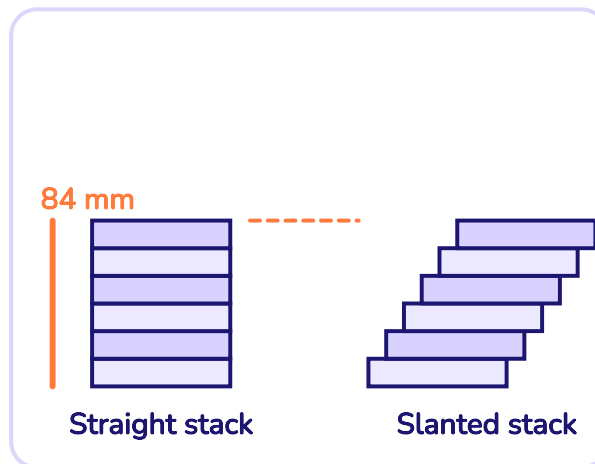
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Example 5: slicing a pyramid

A square pyramid has base side 10 m and height 15 m. A plane parallel to the base is 9 m below the apex, so $k = \frac{9}{15} = 0.6$. The section is a square of side $10 \times 0.6 = 6$ m and area 36 m^2 , which equals $0.6^2 \times 100$.

6. Cavalieri's principle

Why does a leaning stack of coins hold the same amount of metal as a straight one? Slide the coins sideways and nothing changes: each coin keeps its size, and the height stays the same.



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Cavalieri's principle

If two solids lie between the same two parallel planes, and every plane parallel to them cuts both solids in cross sections of equal area, then the two solids have the same volume.

This is why an oblique prism or cylinder has volume $V = Bh$ too, where h is the *perpendicular* height, not the length of the slanted edge. It is also the reason a pyramid's volume depends only on its base area and height, not on where the apex sits above the base.

7. Density and modeling with solids

Density measures how much mass fits in a given volume: $\rho = \frac{m}{V}$, so $m = \rho V$. Common units are grams per cubic centimeter (g/cm^3) or pounds per cubic foot (lb/ft^3). Aluminum is about 2.7 g/cm^3 and steel about 7.85 g/cm^3 .

M Method: modeling a real object

1. Choose solids that approximate the object (a silo is a cylinder plus a hemisphere).
2. Write down the dimensions with units, converting so they match.
3. Add volumes (or subtract a hollow part), then convert units if needed: $1 \text{ L} = 1,000 \text{ cm}^3$, and $1 \text{ ft}^3 \approx 7.48 \text{ gallons}$.
4. Multiply by density to get mass, or compare the result with the real object to judge if it is reasonable.

E Example 6: a solid aluminum ball

A ball of radius 4 cm has $V = \frac{4}{3}\pi \cdot 64 = \frac{256\pi}{3} \approx 268.08 \text{ cm}^3$. With density 2.7 g/cm^3 , its mass is $268.08 \times 2.7 \approx 723.8 \text{ g}$.

! Zyro's tip

On my home planet we say: if every length is multiplied by k , then areas are multiplied by k^2 and volumes by k^3 . Check your answer against that rule before you trust it!

Key takeaways

- Area is in square units; volume is in cubic units; always convert lengths first.
- Prism or cylinder: $V = Bh$. Pyramid or cone: $V = \frac{1}{3}Bh$.
- Sphere: $V = \frac{4}{3}\pi r^3$ and $SA = 4\pi r^2$.
- Use the slant height for the lateral area of cones and pyramids.
- A cross section parallel to the base of a pyramid or cone is similar to the base, with area ratio k^2 .
- Cavalieri: equal heights and equal cross-section areas at every level mean equal volumes.
- Mass equals density times volume.



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