

Area, Surface Area, and Volume: math lesson, Grade 6 – download the PDF

How much paint covers a wall? How much wrapping paper does a gift box need? How much water fits in a fish tank? Each question is about a different measure: **area** covers a flat surface, **surface area** adds up every face of a solid, and **volume** counts how much space is inside. In this chapter you will use rectangles as your building blocks to measure triangles, parallelograms, trapezoids, composite shapes, prisms, and pyramids.

1. Area of triangles

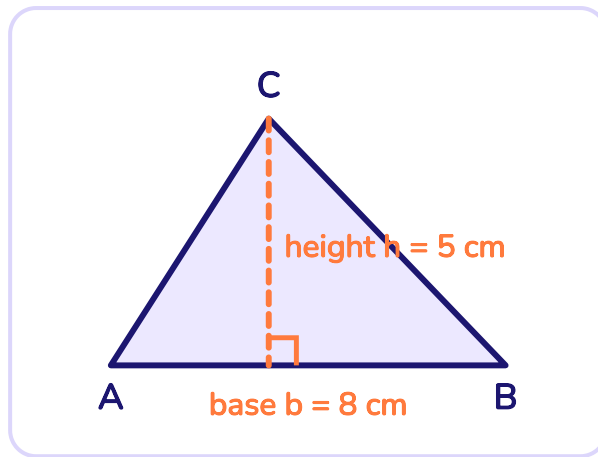
Area is the number of square units that cover a flat shape. Rectangles are easy: $A = \ell \times w$. A triangle is exactly *half* of a rectangle (or parallelogram) with the same base and height. Any side can serve as the **base**, and the **height** is the perpendicular distance from the base to the opposite vertex.

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Area of a triangle

If a triangle has base b and height h (measured at a right angle to the base), then

$$A = \frac{1}{2} b h.$$



E Example 1: a triangular flag

A flag is a triangle with a base of 14 ft and a height of 9 ft.

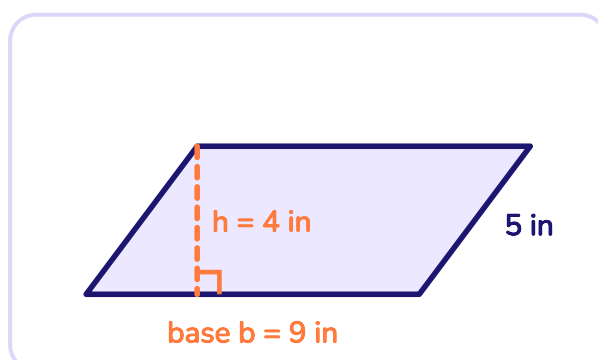
$A = \frac{1}{2} \times 14 \times 9 = 7 \times 9 = 63$. The flag has an area of **63 square feet** (63 ft²).

⚠ Careful with the slanted side

The height must be perpendicular to the base. A slanted side of the triangle is *not* the height, so never multiply by it in the area formula.

2. Area of parallelograms and trapezoids

Cut a triangle off one end of a parallelogram and slide it to the other end: you get a rectangle with the same base and height. So a parallelogram has area $A = b \times h$.

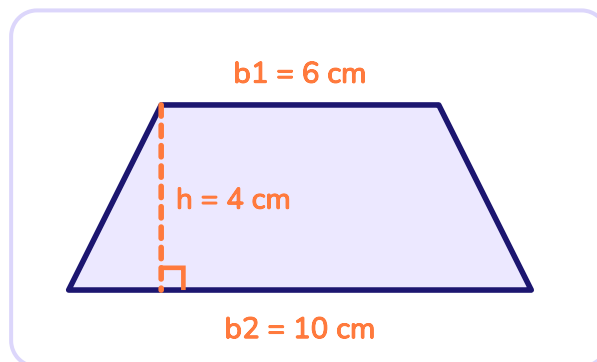


A trapezoid has one pair of parallel sides, called the **bases** b_1 and b_2 . Two identical trapezoids fit together into a parallelogram whose base is $b_1 + b_2$, so one trapezoid is half of it.

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Area formulas

Shape	Area
Parallelogram	$A = b \times h$
Triangle	$A = \frac{1}{2} b h$
Trapezoid	$A = \frac{b_1 + b_2}{2} \times h$



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Example 2: a trapezoid

A trapezoid has bases of 7 cm and 13 cm and a height of 6 cm.

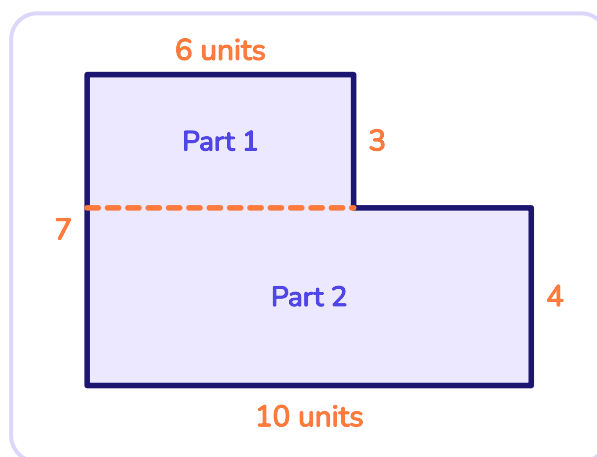
$$A = \frac{7 + 13}{2} \times 6 = 10 \times 6 = 60. \text{ The area is } 60 \text{ cm}^2.$$

3. Area of composite polygons

A **composite polygon** is built from simpler shapes. There are two good strategies: **add** the areas of pieces that fit together, or **subtract** the area of a missing piece from a bigger shape.

M Method: area of a composite shape

1. Draw dashed lines to split the shape into rectangles, triangles, or trapezoids (or to complete it into one big rectangle).
2. Find any missing lengths by adding or subtracting the sides you know.
3. Compute each piece, then add (or subtract).
4. Write the answer with square units.

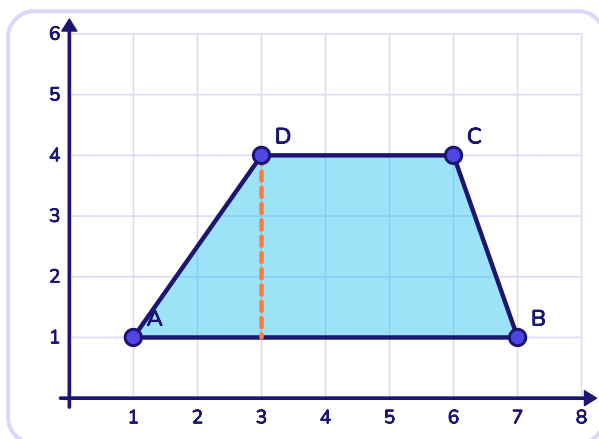


E Example 3: an L-shaped patio

The patio is a 10 by 7 rectangle with a 4 by 3 corner removed. *Adding:* Part 2 is $10 \times 4 = 40$ and Part 1 is $6 \times 3 = 18$, so $A = 40 + 18 = 58$. *Subtracting:* $10 \times 7 - 4 \times 3 = 70 - 12 = 58$. Both methods give **58 square units**, which is a great way to check your work.

4. Polygons on the coordinate plane

When the vertices of a polygon are ordered pairs (x, y) , you can find side lengths by counting grid units. Two points with the same y -coordinate are on a horizontal line, and their distance is the difference of the x -coordinates. Two points with the same x -coordinate are on a vertical line, and the distance is the difference of the y -coordinates. Always take the larger number minus the smaller one; if one coordinate is negative, add the absolute values of the two numbers on opposite sides of zero.



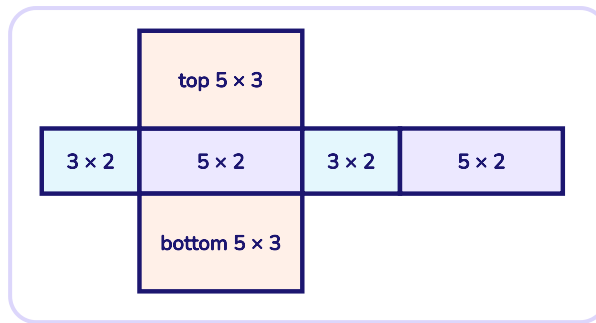
E Example 4: a trapezoid on a grid

Look at trapezoid ABCD with $A(1, 1)$, $B(7, 1)$, $C(6, 4)$, and $D(3, 4)$. Side AB is horizontal: $7 - 1 = 6$. Side DC is horizontal: $6 - 3 = 3$. The height is the vertical distance between the two parallel sides: $4 - 1 = 3$.

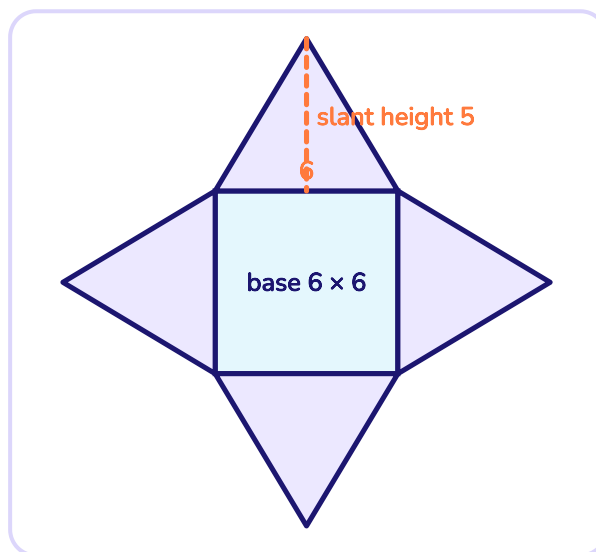
$$A = \frac{6 + 3}{2} \times 3 = 4.5 \times 3 = 13.5. \text{ The area is } 13.5 \text{ square units.}$$

5. Nets of solids

A **net** is a flat pattern that folds up into a solid. The faces of a prism are two congruent **bases** and rectangular **lateral faces**. A pyramid has one base and triangular lateral faces that meet at a point called the apex.



Counting faces tells you the solid. A rectangular prism has 6 rectangles. A triangular prism has 2 triangles and 3 rectangles. A square pyramid has 1 square and 4 triangles.



Zyro's tip

On my home planet we say: “Unfold it, then count.” Sketch the net, label every face with its measurements, and nothing gets forgotten or counted twice!

6. Surface area of prisms and pyramids

The **surface area** of a solid is the sum of the areas of all its faces, which is the area of its net. It is measured in square units.

M**Method: surface area**

1. Sketch the net and list every face.
2. Find the area of each face.
3. Add them all up (use symmetry: matching faces have equal areas).
4. Give the answer in square units.

E**Example 5: a rectangular prism**

A box is 5 in long, 3 in wide, and 2 in tall. The faces come in three matching pairs:

$5 \times 3 = 15$, $5 \times 2 = 10$, and $3 \times 2 = 6$. So $SA = 2(15 + 10 + 6) = 2 \times 31 = 62$.

The surface area is **62 in²**.

E**Example 6: a square pyramid**

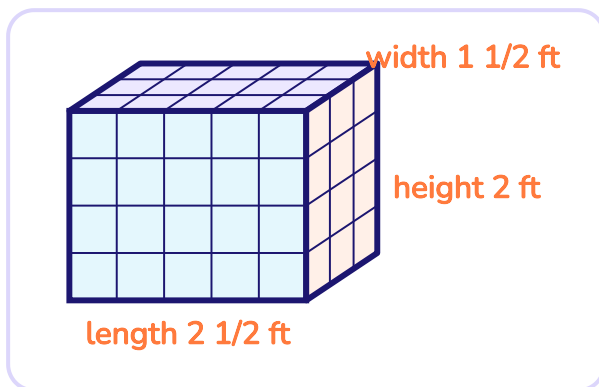
A pyramid has a square base of side 6 in and four triangular faces, each with base 6 in and slant height 5 in. The base is $6 \times 6 = 36$. Each triangle is $\frac{1}{2} \times 6 \times 5 = 15$. So $SA = 36 + 4 \times 15 = 96$. The surface area is **96 in²**.

**Slant height is not height**

For the triangular faces of a pyramid, use the *slant height* (the height of the triangle), not the vertical height of the whole pyramid.

7. Volume of rectangular prisms with fractional edges

Volume measures the space inside a solid in **cubic units**. Fill a prism with unit cubes and count them: the volume of a rectangular prism is $V = \ell \times w \times h$, or $V = B \times h$ where B is the area of the base. This still works when the edges are fractions or mixed numbers.



E Example 7: fractional edges

A prism measures $2\frac{1}{2}$ ft by $1\frac{1}{2}$ ft by 2 ft. Pack it with cubes of edge $\frac{1}{2}$ ft: there are $5 \times 3 \times 4 = 60$ cubes. Each cube has volume $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ ft³, so $V = 60 \times \frac{1}{8} = 7.5$ ft³. The formula agrees: $\frac{5}{2} \times \frac{3}{2} \times 2 = \frac{30}{4} = 7.5$. The volume is **7.5 cubic feet**.



Three different units

Length is in cm, area in cm², volume in cm³. Mixing them up is one of the most common mistakes, so check the unit before you write your answer.

Key takeaways

- Triangle: $A = \frac{1}{2}bh$; parallelogram: $A = bh$; trapezoid: $A = \frac{b_1 + b_2}{2}h$. The height is always perpendicular to the base.
- For a composite polygon, split it into simple shapes and add, or subtract a missing piece.
- On the coordinate plane, subtract matching coordinates to get horizontal and vertical lengths.
- A net is a solid unfolded flat; count its faces to name the solid.
- Surface area = sum of the areas of all faces (square units). Use slant height for pyramid triangles.
- Volume of a rectangular prism: $V = \ell \times w \times h$, even with fractional edges (cubic units).



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