

# Applications of Integration: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

## 1 Area between a line and a parabola



Solve  $x = x^2$ :  $x = 0$  or  $x = 1$ . On  $(0, 1)$  the line is above the parabola (try  $x = \frac{1}{2}$ :  $0.5 > 0.25$ ).

$$A = \int_0^1 (x - x^2) dx = \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

The area is  $\frac{1}{6}$  square unit.

## 2 Area under a roof line



The curves meet when  $x^2 = 4$ , so  $x = \pm 2$ . The top curve is  $y = 4$ .

$$A = \int_{-2}^2 (4 - x^2) dx = \left[ 4x - \frac{x^3}{3} \right]_{-2}^2 = \frac{16}{3} + \frac{16}{3} = \frac{32}{3}.$$

The window has area  $\frac{32}{3} \approx 10.67 \text{ ft}^2$ .

### 3 A cone from a line



Each disk has radius  $x$ :  $V = \pi \int_0^3 x^2 dx = \pi \cdot \frac{27}{3} = 9\pi$ .

Check: the solid is a cone with radius 3 and height 3, so  $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(9)(3) = 9\pi$ .

The volume is  $9\pi \approx 28.27$  cubic units.

### 4 Length of a ramp



$f'(x) = 3$ , so  $L = \int_0^4 \sqrt{1+9} dx = 4\sqrt{10} \approx 12.65$ .

Check: the endpoints are  $(0, 1)$  and  $(4, 13)$ , so the distance is

$\sqrt{4^2 + 12^2} = \sqrt{160} = 4\sqrt{10}$ . Both agree.

### 5 Average of a linear function



$\int_0^4 (2x + 1) dx = [x^2 + x]_0^4 = 20$ , so  $f_{\text{avg}} = \frac{20}{4} = 5$ .

Solve  $2x + 1 = 5$ :  $x = 2$ , the midpoint, as expected for a straight line.

### 6 Constant force



$W = \int_0^{12} 25 dx = 25 \cdot 12 = 300$  ft·lb.

In joules:  $300 \times 1.3558 \approx 406.7$  J.

### 7 Checking a differential equation



$y' = -2Ce^{-2x} = -2y$ , so every  $C$  works.

$y(0) = C = 5$ , so  $y = 5e^{-2x}$  and  $y(1) = 5e^{-2} \approx 0.677$ .

### 8 Square root against a line



$\sqrt{x} = \frac{x}{2}$  gives  $x = 0$  or  $x = 4$ . On  $(0, 4)$  the square root is on top (at  $x = 1$ :  $1 > 0.5$ ).

$$A = \int_0^4 \left(x^{1/2} - \frac{x}{2}\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{x^2}{4}\right]_0^4 = \frac{16}{3} - 4 = \frac{4}{3}.$$

### 9 Curves that cross



$x = x^3$  gives  $x = -1, 0, 1$ . On  $(0, 1)$  the line is on top; on  $(-1, 0)$  the cubic is on top, so we must split.

$$\text{By symmetry } A = 2 \int_0^1 (x - x^3) dx = 2\left(\frac{1}{2} - \frac{1}{4}\right) = \frac{1}{2}.$$

### 10 Washer about the x-axis



The curves meet at  $x = 0$  and  $x = 2$ ;  $2x$  is the outer radius and  $x^2$  the inner radius.

$$V = \pi \int_0^2 (4x^2 - x^4) dx = \pi \left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi}{15} \approx 13.40.$$

### 11 A cubic bowl



$$V = \pi \int_0^1 x^6 dx = \pi \cdot \frac{1}{7} = \frac{\pi}{7} \approx 0.449 \text{ cubic unit.}$$

### 12 A vase by shells



Radius  $x$ , height  $4x - x^2$ :

$$V = 2\pi \int_0^4 (4x^2 - x^3) dx = 2\pi \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^4 = 2\pi \left(\frac{256}{3} - 64\right) = \frac{128\pi}{3} \approx 134.0.$$

**13 Length of a curved cable**

★★★

$$y' = \frac{3}{2}\sqrt{x}, \text{ so } 1 + (y')^2 = 1 + \frac{9}{4}x.$$

$$L = \int_0^4 \sqrt{1 + \frac{9}{4}x} \, dx = \frac{8}{27} \left[ \left(1 + \frac{9}{4}x\right)^{3/2} \right]_0^4 = \frac{8}{27} (10^{3/2} - 1) \approx 9.07 \text{ ft.}$$

**14 Stretching a spring in metric units**

★★★

$$W = \int_{0.1}^{0.3} 200x \, dx = 100x^2 \Big|_{0.1}^{0.3} = 100(0.09 - 0.01) = 8 \text{ J.}$$

**15 Average temperature**

★★★

$$\int_0^{12} \sin\left(\frac{\pi t}{12}\right) dt = \frac{12}{\pi} \left[ -\cos \frac{\pi t}{12} \right]_0^{12} = \frac{24}{\pi}.$$

$$T_{\text{avg}} = 60 + \frac{20}{12} \cdot \frac{24}{\pi} = 60 + \frac{40}{\pi} \approx 72.7^\circ\text{F (about } 22.6^\circ\text{C)}.$$

**16 Integrating with respect to  $y$** 

★★★★

Solve  $y^2 = y + 6$ :  $(y - 3)(y + 2) = 0$ , so  $y = -2$  or  $y = 3$ . The line is to the right of the parabola in between.

$$A = \int_{-2}^3 (y + 6 - y^2) \, dy = \left[ \frac{y^2}{2} + 6y - \frac{y^3}{3} \right]_{-2}^3 = \frac{27}{2} + \frac{22}{3} = \frac{125}{6} \approx 20.83.$$

**17 Shells about a vertical line**

★★★★

A strip at  $x$  is at distance  $2 - x$  from the axis and has height  $x^2$ .

$$V = 2\pi \int_0^1 (2 - x)x^2 \, dx = 2\pi \left( \frac{2}{3} - \frac{1}{4} \right) = \frac{5\pi}{6} \approx 2.62.$$

**18 Washers about a horizontal line**

★★★

The distances to the axis are  $R = x + 1$  and  $r = x^2 + 1$ .

$$V = \pi \int_0^1 [(x+1)^2 - (x^2+1)^2] dx = \pi \int_0^1 (2x - x^2 - x^4) dx = \pi \left(1 - \frac{1}{3} - \frac{1}{5}\right) = \frac{7\pi}{15}$$

**19 An arc that needs a table**

★★★

$$y' = x, \text{ so } L = \int_0^1 \sqrt{1+x^2} dx = \frac{1}{2} (\sqrt{2} + \ln(1+\sqrt{2})) \approx 1.148.$$

**20 Pumping water from a tank**

★★★

A layer at height  $y$  above the bottom has volume  $9\pi dy$ , weighs  $62.4 \cdot 9\pi dy$  pounds and is lifted  $8 - y$  feet.

$$W = \int_0^8 62.4 \cdot 9\pi (8 - y) dy = 561.6\pi \cdot 32 = 17,971.2\pi \approx 56,458 \text{ ft}\cdot\text{lb}.$$

**21 A separable initial-value problem**

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Separate:  $e^y dy = x dx$ . Integrate:  $e^y = \frac{x^2}{2} + C$ . From  $y(0) = 0$ :  $1 = C$ .

So  $y = \ln\left(1 + \frac{x^2}{2}\right)$  and  $y(2) = \ln 3 \approx 1.099$ .

**22 Cooling coffee**

★★★

Separate:  $\frac{dT}{T-70} = -k dt$ , so  $T - 70 = Ae^{-kt}$  with  $A = 120$ .

At  $t = 10$ :  $80 = 120e^{-10k}$ , so  $e^{-10k} = \frac{2}{3}$ . At  $t = 20$ :  $T = 70 + 120 \cdot \frac{4}{9} = \frac{370}{3} \approx 123.3^\circ\text{F}$  (about  $50.7^\circ\text{C}$ ).

For  $T = 100$ :  $30 = 120(2/3)^{t/10}$ , so  $t = 10 \frac{\ln 4}{\ln 1.5} \approx 34.2$  minutes.



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