

Applications of Derivatives: practice solutions, College – download the PDF

Written solutions to the chapter problems. Check each step, then correct yourself.

1 Where does it rise?



$f'(x) = 2x - 6 = 0$ gives $x = 3$. For $x < 3$, $f' < 0$: decreasing on $(-\infty, 3)$. For $x > 3$, $f' > 0$: increasing on $(3, \infty)$.

The sign changes from $-$ to $+$, so $f(3) = 9 - 18 + 5 = -4$ is a local (and absolute) minimum.

2 True or false?



(a) **True**: a positive derivative means positive tangent slopes, so f increases on the whole interval.

(b) **False**: take $f(x) = (x - 2)^3$. Then $f'(2) = 0$, but f is increasing on both sides of 2 , so there is no extremum.

3 A cubic hill and valley



$g'(x) = 3x^2 - 12 = 3(x - 2)(x + 2)$, so the critical points are $x = -2$ and $x = 2$.

$g' > 0$ for $x < -2$, $g' < 0$ on $(-2, 2)$, $g' > 0$ for $x > 2$. So $g(-2) = -8 + 24 = 16$ is a local maximum and $g(2) = 8 - 24 = -16$ is a local minimum.

4 Concavity check



$f'(x) = 3x^2 - 18x$ and $f''(x) = 6x - 18$, which is zero at $x = 3$.

$f'' < 0$ for $x < 3$: concave down; $f'' > 0$ for $x > 3$: concave up. The concavity changes, so the inflection point is $(3, f(3)) = (3, 27 - 81 + 2) = (3, -52)$.

5 Estimating a square



$f(10) = 100$ and $f'(10) = 20$, so $L(x) = 100 + 20(x - 10)$.

$L(10.2) = 100 + 20(0.2) = 104$. The exact value is $10.2^2 = 104.04$, so the error is **0.04**.

6 A hard-looking limit



At $x = 3$ the form is $\frac{0}{0}$. So the limit equals $\lim_{x \rightarrow 3} \frac{2x}{1} = 6$.

Check: $\frac{x^2 - 9}{x - 3} = x + 3$ for $x \neq 3$, which tends to 6.

7 A growing square



$A = s^2$, so $\frac{dA}{dt} = 2s \frac{ds}{dt}$.

When $s = 10$: $\frac{dA}{dt} = 2(10)(3) = 60 \text{ cm}^2/\text{s}$.

8 The open box



Base side $12 - 2x$, height x : $V(x) = x(12 - 2x)^2 = 4x^3 - 48x^2 + 144x$ with $x \in (0, 6)$.

$V'(x) = 12x^2 - 96x + 144 = 12(x - 2)(x - 6)$. Only $x = 2$ lies in $(0, 6)$. V increases before 2 and decreases after, and V is 0 at both ends.

The maximum volume is $V(2) = 2 \cdot 8^2 = 128 \text{ in}^3$ (about **2,097 cm³**), for $x = 2$ in.

9 Bending quartic



$$f'(x) = 4x^3 - 12x, f''(x) = 12x^2 - 12 = 12(x-1)(x+1).$$

$f'' > 0$ for $x < -1$ and $x > 1$ (concave up); $f'' < 0$ on $(-1, 1)$ (concave down). Inflection points: $(-1, -5)$ and $(1, -5)$, since $f(\pm 1) = 1 - 6 = -5$.

10 Mean Value Theorem for a cube



f is a polynomial, so it is continuous on $[0, 3]$ and differentiable on $(0, 3)$.

The average rate is $\frac{27-0}{3-0} = 9$. Solve $3c^2 = 9$: $c = \pm\sqrt{3}$. Only $c = \sqrt{3} \approx 1.732$ lies in $(0, 3)$.

11 The turnpike driver



Position $s(t)$ is continuous and differentiable on the 2-hour interval. The average speed is $\frac{170-20}{2} = 75$ mph.

By the Mean Value Theorem, there is a time c between 1:00 pm and 3:00 pm with $s'(c) = 75$: the speedometer read exactly 75 mph (about 120.7 km/h) at that moment.

12 Twice L'Hôpital



At $x = 0$ the form is $\frac{0}{0}$. First application: $\frac{2e^{2x} - 2}{2x}$, still $\frac{0}{0}$. Second application: $\frac{4e^{2x}}{2} \rightarrow \frac{4}{2} = 2$.

The limit is 2.

13 The sliding ladder

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$x^2 + y^2 = 100$, so $x \frac{dx}{dt} + y \frac{dy}{dt} = 0$. With $x = 6$, $y = 8$: $\frac{dy}{dt} = -\frac{6}{8}(2) = -1.5$ ft/s.

The top moves down at 1.5 ft/s; the negative sign means y is decreasing.

14 A cube root estimate

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$f(27) = 3$ and $f'(x) = \frac{1}{3}x^{-2/3}$, so $f'(27) = \frac{1}{3 \cdot 9} = \frac{1}{27}$.

$L(28) = 3 + \frac{1}{27} \approx 3.0370$. The calculator value is **3.0366**, so the estimate is excellent.

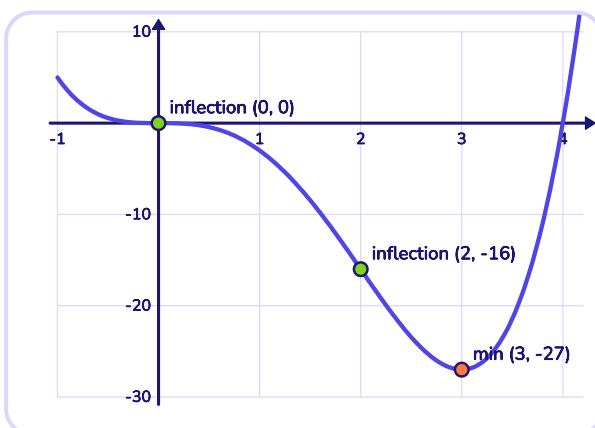
15 The quartic that fooled the test

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$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$: critical points $x = 0$ and $x = 3$. $f' < 0$ for $x < 3$ except at 0 (where it is 0), and $f' > 0$ for $x > 3$. So there is no extremum at 0, and $f(3) = 81 - 108 = -27$ is a local minimum.

$f''(x) = 12x^2 - 24x = 12x(x - 2)$: positive for $x < 0$, negative on $(0, 2)$, positive for $x > 2$. Inflection points: $(0, 0)$ and $(2, -16)$.

At $x = 0$ the second derivative test fails because $f''(0) = 0$; the first derivative test shows the sign of f' does not change.



16 Closest point on a curve

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Minimize the squared distance $D(x) = (x - 4)^2 + (\sqrt{x})^2 = (x - 4)^2 + x$ for $x \geq 0$.

$D'(x) = 2(x - 4) + 1 = 2x - 7 = 0$ gives $x = 3.5$. D decreases before and increases after, and $D(0) = 16$ is larger. $D(3.5) = 0.25 + 3.5 = 3.75$.

The closest point is $(3.5, \sqrt{3.5}) \approx (3.5, 1.871)$, at distance $\sqrt{3.75} \approx 1.94$.

17 The cheapest can

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$\pi r^2 h = 500$ gives $h = \frac{500}{\pi r^2}$. Then $S(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{1000}{r}$.

$S'(r) = 4\pi r - \frac{1000}{r^2} = 0$ gives $r^3 = \frac{250}{\pi}$, so $r = \sqrt[3]{250/\pi} \approx 4.30$ cm. Since

$S''(r) = 4\pi + \frac{2000}{r^3} > 0$, this is a minimum.

Then $h = \frac{500}{\pi r^2} \approx 8.60$ cm, which is $2r$: the best can is as tall as it is wide.

18 Filling a cone

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By similar triangles $r = \frac{h}{2}$, so $V = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{12}$.

$\frac{dV}{dt} = \frac{\pi h^2}{4} \frac{dh}{dt}$. At $h = 4$: $4 = \frac{16\pi}{4} \frac{dh}{dt} = 4\pi \frac{dh}{dt}$, so $\frac{dh}{dt} = \frac{1}{\pi} \approx 0.318$ ft/min (about 9.7 cm/min).

19 Two indeterminate forms

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(a) $x^2 e^{-x} = \frac{x^2}{e^x}$, form $\frac{\infty}{\infty}$. Apply twice: $\frac{2x}{e^x}$, then $\frac{2}{e^x} \rightarrow 0$. The limit is 0.

(b) $x \ln x = \frac{\ln x}{1/x}$, form $\frac{-\infty}{\infty}$. Then $\frac{1/x}{-1/x^2} = -x \rightarrow 0$. The limit is 0.

20 An inequality from the MVT



f is continuous on $[0, x]$ and differentiable on $(0, x)$. The MVT gives $c \in (0, x)$ with
$$\frac{\ln(1+x) - \ln 1}{x} = f'(c) = \frac{1}{1+c}.$$

Since $c > 0$, $\frac{1}{1+c} < 1$, so $\ln(1+x) < x$.

Check: $\ln 1.5 \approx 0.405 < 0.5$.

21 Find the cubic



$f'(x) = 3x^2 + 2ax + b$ must vanish at -1 and 3 , so

$f'(x) = 3(x+1)(x-3) = 3x^2 - 6x - 9$. Hence $a = -3$ and $b = -9$, and

$f(x) = x^3 - 3x^2 - 9x$.

$f(-1) = -1 - 3 + 9 = 5$ (local maximum) and $f(3) = 27 - 27 - 27 = -27$ (local minimum).

$f''(x) = 6x - 6$ changes sign at $x = 1$; $f(1) = 1 - 3 - 9 = -11$, so the inflection point is $(1, -11)$.



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