

Applications of Derivatives: math practice, College – download the PDF

21 problems graded by difficulty (★ easy, ★★ medium, ★★★ hard). Try first, without looking at the solutions!

1 Where does it rise?

★ ★ ★

Let $f(x) = x^2 - 6x + 5$. Find f' , the critical point, and the intervals where f is increasing or decreasing. Identify the extremum.

2 True or false?

★ ★ ★

Decide and justify. (a) If $f'(x) > 0$ on (a, b) , then f is increasing there. (b) If $f'(2) = 0$, then f has a local extremum at $x = 2$.

3 A cubic hill and valley

★ ★ ★

Let $g(x) = x^3 - 12x$. Find the critical points and classify them with the first derivative test.

4 Concavity check

★ ★ ★

For $f(x) = x^3 - 9x^2 + 2$, find where the graph is concave up, concave down, and its inflection point.

5 Estimating a square



Use the linear approximation of $f(x) = x^2$ at $a = 10$ to estimate 10.2^2 . How far is the estimate from the exact value?

6 A hard-looking limit



Compute $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$ using L'Hôpital's rule, and check by factoring.

7 A growing square



The side s of a square tile grows at 3 cm/s. How fast is its area increasing when $s = 10$ cm?

8 The open box



A baker cuts four equal squares of side x inches from the corners of a 12 in by 12 in sheet of cardboard and folds up the sides to make an open box. Find x that maximizes the volume, and that volume.

9 Bending quartic



Let $f(x) = x^4 - 6x^2$. Find the intervals of concavity and the inflection points.

10 Mean Value Theorem for a cube



Show that $f(x) = x^3$ satisfies the hypotheses of the Mean Value Theorem on $[0, 3]$ and find all values c it guarantees.

11 The turnpike driver



A driver enters a highway at mile marker 20 at 1:00 pm and exits at mile marker 170 at 3:00 pm. Show that the car traveled at exactly 75 mph at some moment. (Assume position is a differentiable function of time.)

12 Twice L'Hôpital



Compute $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - 2x}{x^2}$.

13 The sliding ladder



A 10 ft ladder leans on a wall. Its foot slides away at 2 ft/s. How fast is the top moving down when the foot is 6 ft from the wall? What is the sign of the answer?

14 A cube root estimate



Use a linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 27$ to estimate $\sqrt[3]{28}$.

15 The quartic that fooled the test



Study $f(x) = x^4 - 4x^3$: critical points, extrema, concavity, and inflection points. Why does $f''(0) = 0$ not decide anything about $x = 0$?

16 Closest point on a curve



Find the point on the curve $y = \sqrt{x}$ that is closest to $(4, 0)$, and the minimal distance.

17 The cheapest can

★★★

A closed cylindrical can must hold 500 cm^3 . Find the radius and height that minimize the metal used (surface area), to the nearest hundredth.

18 Filling a cone

★★★

Water pours at $4 \text{ ft}^3/\text{min}$ into a conical tank with the point down, radius 3 ft and height 6 ft at the top. How fast is the water level rising when the depth is 4 ft?

19 Two indeterminate forms

★★★

Compute (a) $\lim_{x \rightarrow \infty} x^2 e^{-x}$ and (b) $\lim_{x \rightarrow 0^+} x \ln x$. (Rewrite each as a quotient first.)

20 An inequality from the MVT

★★★

Use the Mean Value Theorem on $f(t) = \ln(1+t)$ over $[0, x]$, with $x > 0$, to prove that $\ln(1+x) < x$.

21 Find the cubic

★★★

The function $f(x) = x^3 + ax^2 + bx$ has a local maximum at $x = -1$ and a local minimum at $x = 3$. Find a and b , the two extreme values, and the inflection point.



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