

Angles and Geometric Constructions: math lesson, Grade 7 – download the PDF

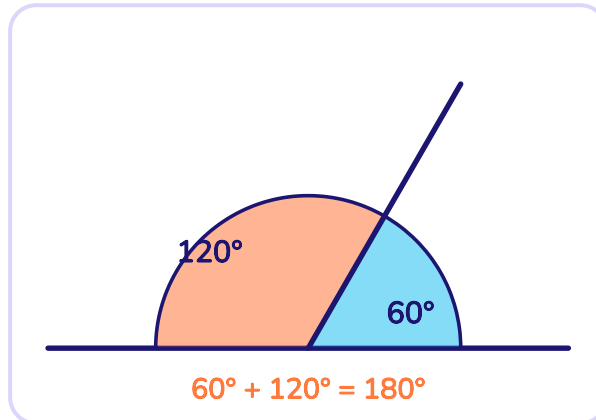
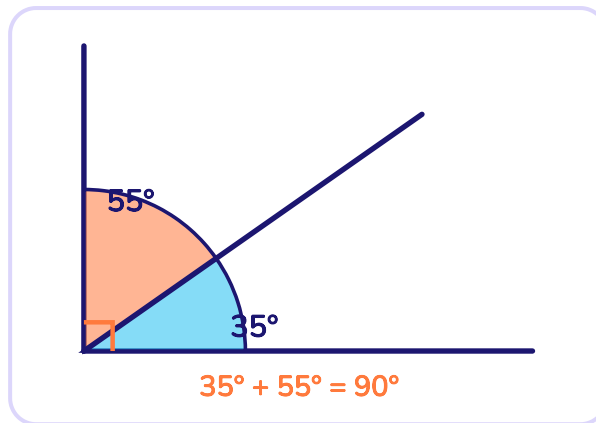
Look at the corner of a door, the hands of a clock, or the legs of a folding chair. Angles are everywhere, and they follow rules that let you find a missing measure without a protractor. In this chapter you will learn those rules, write equations to solve for unknown angles, decide when three lengths can build a triangle, draw triangles with a ruler and a protractor, and picture the flat shapes you see when you slice a solid.

1. Complementary and supplementary angles

D Definitions

Two angles are **complementary** if their measures add up to 90° . Two angles are **supplementary** if their measures add up to 180° .

The two angles do not have to touch. Any two angles with the right total are a pair. If an angle measures a° , then its complement is $(90 - a)^\circ$ and its supplement is $(180 - a)^\circ$.



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Example 1

Find the complement and the supplement of 62° .

Complement: $90 - 62 = 28$, so 28° . Supplement: $180 - 62 = 118$, so 118° .



Watch out

An angle larger than 90° has no complement, because the other angle would need a negative measure. Every angle smaller than 180° has a supplement.

2. Adjacent angles

D Definition

Two angles are **adjacent** when they share a vertex and one side, and they do not overlap. Their other sides are on opposite sides of the shared ray.

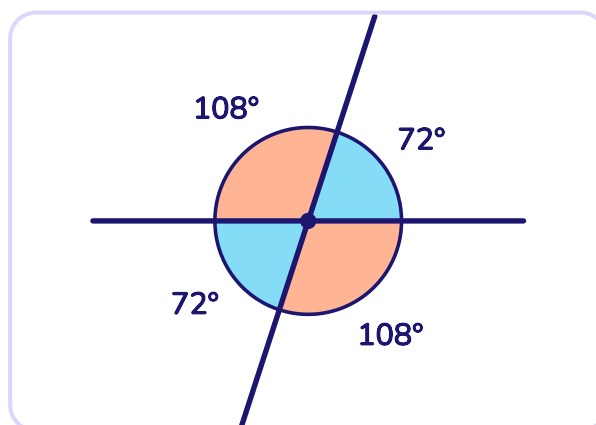
Adjacent angles can be added to give the measure of the big angle they form together. Two special cases are very useful:

- if the outer sides form a right angle, the two adjacent angles are complementary;
- if the outer sides form a straight line, the two adjacent angles are supplementary. We call them a **linear pair**.

The two angles in the figures above are adjacent: they share a vertex and the middle ray.

3. Vertical angles

When two lines cross, they form four angles. The angles that sit across from each other are **vertical angles**.



P Property

Vertical angles are congruent: they have the same measure. Each angle and its neighbor form a linear pair, so they add up to 180° .

Why is it true? Call the angles around the crossing a, b, c, d in order. Then $a + b = 180^\circ$ and $c + b = 180^\circ$. So $a + b = c + b$, which gives $a = c$.

I 4. Writing equations to find unknown angles

M Method

1. Decide which relationship links the angles: complementary (total 90°), supplementary or linear pair (total 180°), vertical (equal), or a full turn (total 360°).
2. Write an equation using the expressions given.
3. Solve the equation for the variable.
4. Substitute back to find each angle, and check that the relationship holds.

E Example 2

Two vertical angles measure $(3x + 10)^\circ$ and $(5x - 20)^\circ$. Find x and the angle.

Vertical angles are equal: $3x + 10 = 5x - 20$. Then $30 = 2x$, so $x = 15$. The angle is $3(15) + 10 = 55$, so each of the two angles measures 55° . The other two angles each measure $180 - 55 = 125$, so 125° .

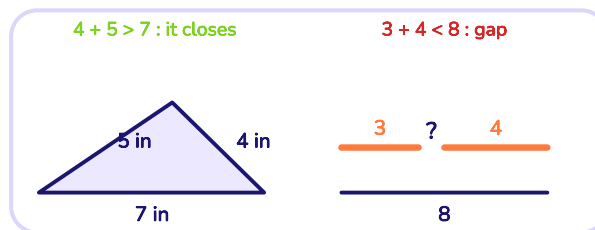
E**Example 3**

Two angles are complementary. The second is 9° more than twice the first. Find both.

Let x be the first angle. Then $x + (2x + 9) = 90$, so $3x = 81$ and $x = 27$. The angles are 27° and $2(27) + 9 = 63^\circ$. Check: $27 + 63 = 90$.

5. The triangle inequality

You cannot build a triangle from just any three lengths. Imagine two sticks that are too short to reach across the third side: they never meet.

**P****Triangle inequality**

Three lengths can be the sides of a triangle if and only if the sum of the two shorter lengths is greater than the longest length. Equivalently, every side is shorter than the sum of the other two.

If the sum is exactly equal to the longest side, the shorter sides lie flat on it and the triangle collapses into a segment.

E**Example 4**

Can 4 in, 6 in and 9 in be the sides of a triangle? Can 3 in, 5 in and 9 in?

For 4, 6, 9: $4 + 6 = 10 > 9$, so yes. For 3, 5, 9: $3 + 5 = 8 < 9$, so no.

E**Example 5**

Two sides of a triangle are 7 cm and 12 cm. What can the third side s be?

The third side must be less than the sum: $s < 7 + 12 = 19$. The long side 12 must also be less than $7 + s$, so $s > 12 - 7 = 5$. Therefore $5 < s < 19$.

6. Conditions for a unique triangle

Sometimes the given information determines exactly one triangle, sometimes none, and sometimes many.

- **Three side lengths** that satisfy the triangle inequality give exactly one triangle (up to turning or flipping it). If they fail the inequality, there is no triangle.
- **Two sides and the angle between them** give exactly one triangle.
- **Two angles and the side between them** give exactly one triangle.
- **Three angles only** give many triangles: same shape, different sizes. The angles must also add up to 180° , or no triangle exists.

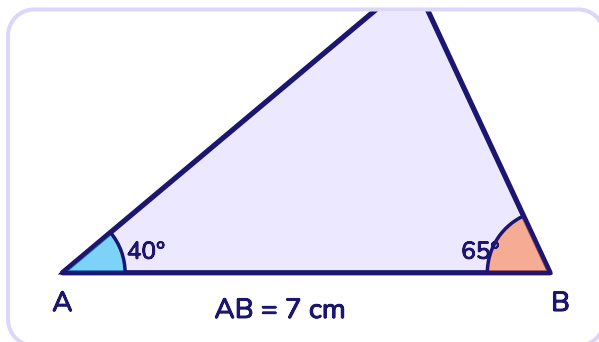
! Zyro's tip

On my planet we say: lengths pin down the size, angles pin down the shape. To get one exact triangle you need at least one length.

7. Drawing triangles with a ruler and a protractor

M Method: draw a triangle from one side and two angles

1. Draw segment AB with the ruler at the required length.
2. Place the center of the protractor on A with its baseline along AB . Mark the first angle and draw a long ray from A .
3. Do the same at B , making sure to measure from side BA .
4. Label C , the point where the two rays cross.



E Example 6

Draw a triangle with $AB = 7 \text{ cm}$, $\angle A = 40^\circ$ and $\angle B = 65^\circ$. What is $\angle C$?

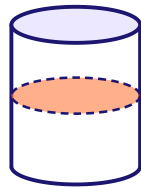
The angles of a triangle add up to 180° , so $\angle C = 180 - 40 - 65 = 75^\circ$. You can check this by measuring $\angle C$ on your drawing: it should be within one degree.

To draw a triangle from two sides and the angle between them, draw the first side, measure the angle at one end, and mark the second length along the new ray with the ruler. Then join the two free ends.

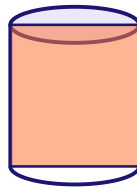
8. Cross sections of three-dimensional figures

D Definition

A **cross section** is the two-dimensional shape you get when a plane slices through a solid.



parallel cut : circle



cut through axis : rectangle

Solid	Cut	Cross section
Rectangular prism	parallel to a face	rectangle the same size as that face
Rectangular prism	perpendicular to the base	rectangle
Cylinder	parallel to the bases	circle
Cylinder	through the axis	rectangle
Cone	parallel to the base	circle
Cone	through the tip, perpendicular to the base	triangle
Sphere	any plane	circle
Square pyramid	parallel to the base	square

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Example 7

A rectangular prism is 10 in long, 6 in wide and 4 in tall. A cut parallel to the top face makes what shape, and what is its area?

The cross section is a rectangle 10 in by 6 in. Its area is $10 \times 6 = 60 \text{ in}^2$.

Key takeaways

- Complementary angles add to 90° ; supplementary angles add to 180° .
- Adjacent angles share a vertex and a side; angles on a straight line add to 180° .
- Vertical angles are equal.
- To find unknown angles, name the relationship, write an equation, solve, then check.
- Triangle inequality: the two shorter sides must add up to more than the longest side.
- Three valid sides, or two sides and the included angle, or two angles and the included side give exactly one triangle.
- A cross section is the flat shape made by a plane cutting a solid; it depends on the direction of the cut.



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